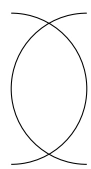
Math 3385: Homework #8 answers

#6.9a

Assume A is connected in X, and to get a contradiction also assume that A is not a subset of some component. That means there is a component $C \subseteq X$ such that some points of A are in C, and some points from A are not in C. That is, $A \cap C \neq \emptyset$ and $A \cap (X - C) \neq \emptyset$. And clearly $A \cap C \cap (X - C) = \emptyset$. All this together means that C and $(X - C)$ make a separation of A, and so A is disconnected which contradicts our original assumption.

#6.18

(a) Something like this, where A and B are the two semicircular arcs:



Then $A \cap B$ is two disconnected points.
(b) Use the same example as above. In this case $A - B$ is one of the arcs but with two points deleted, and this will be disconnected.
(c) In $\mathbb{R}$, let $A = [0, 1]$ and $B = [0, 1] \cup [2, 3]$.
(d) In $\mathbb{R}$, let $A = [0, 1] \cup [2, 3]$, and $B = [-1, 0] \cup [1, 2]$.
(e) In $\mathbb{R}$, let $A = (0, 1)$ and $B = (1, 2)$. Then $\bar{A} \cap \bar{B} = \{1\}$ which is not empty, but $A \cup B$ is disconnected.

#6.38a

Let $g(x) = f(x) - f(A(x))$, and we will show that $g(x) = 0$ for some x. Note that $g(A(x)) = f(A(x)) - f(x) = -(f(x) - f(A(x))) = -g(x)$, so if $g(x) > 0$ then $g(A(x)) < 0$. Thus g attains some positive values, and some negative values, and since it is continuous, this means that g attains the value 0.

#6.46

Assume that A_1 and A_2 are path connected, and $A_1 \cap A_2 \neq \emptyset$. We need to show that $A_1 \cup A_2$ is path connected, so take $a, b \in A_1 \cup A_2$, and we'll show there is a path from a to b.

Let $c \in A_1 \cap A_2$. Then since A_1 and A_2 are each path connected, there are paths from a to c and also from c to b. Thus we can stitch those paths together and we get a path from a to b as desired.