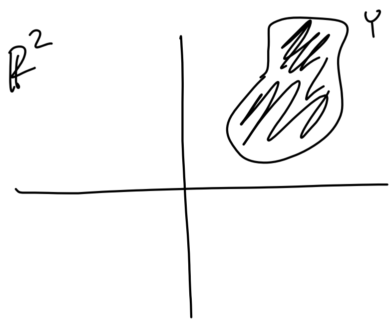
Building new top. spaces.

Easist to create a <u>subspace</u>



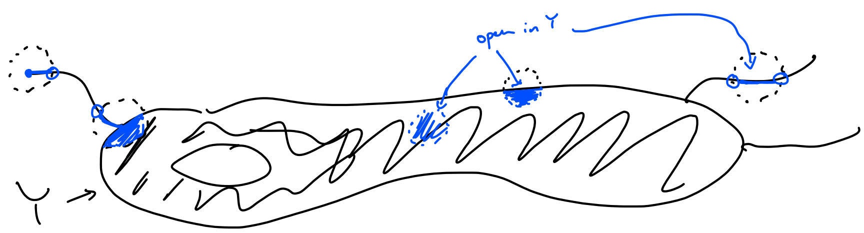
Y can inherit the topology from $\mathbb{R}^2$ to become its own space.

<u>Def</u> Let X be a top. space, and $Y \subset X$.

Then $\mathcal{T}_Y = \{ U \cap Y \mid U \text{ is open in } X \}$

This is a topology on Y, called <u>the subspace topology</u>

Sets in $\mathcal{T}_Y$ are called "open in Y"



subspace of $\mathbb{R}^2$

<u>Thm</u> $\mathcal{T}_Y = \{ U \cap Y \mid U \text{ is open in } X \}$ is a topology on Y.

<u>Pf</u> <u>i</u> ($\emptyset$ & Y are open)

$\emptyset = \overset{\text{open in } X}{\emptyset} \cap Y$, so $\emptyset$ is open in Y.

$$Y = \underbrace{X}_{\text{open in } X} \cap Y$$ so Y is open in Y.

$\underline{ii}$ (Fin. $\cap$s) Let $U_1, \dots, U_n$ be open in Y,

WTS $\bigcap_{i=1}^{n} U_i$ is open in Y.

Since U_i is open in Y, we have

$U_i = V_i \cap Y$ for V_i open in X.

So $$\bigcap_{i=1}^{n} U_i = \bigcap_{i=1}^{n} V_i \cap Y$$

$$= \underbrace{\left(\bigcap_{i=1}^{n} V_i\right)}_{\text{open in } X} \cap Y$$ So this is open in Y.

$\underline{iii}$ Any unions: Let U_α be open in Y for all $\alpha \in A$.

WTS $\bigcup_{\alpha \in A} U_\alpha$ is open in Y.

Since U_α is open in Y,

we have $U_\alpha = V_\alpha \cap Y$ for V_α open in X.

Then $$\bigcup_{\alpha \in A} U_\alpha = \bigcup_{\alpha \in A} V_\alpha \cap Y$$

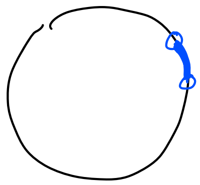
$$= \left(\bigcup_{\alpha \in A} V_\alpha \right) \cap Y$$

$\uparrow$

open in X

So this is open in Y.

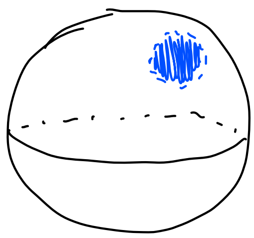
Y:



What does a basis nbhd look like in $Y \subseteq \mathbb{R}^2$?

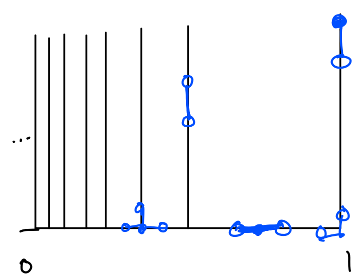
Like a small open interval.

in $\mathbb{R}^3$



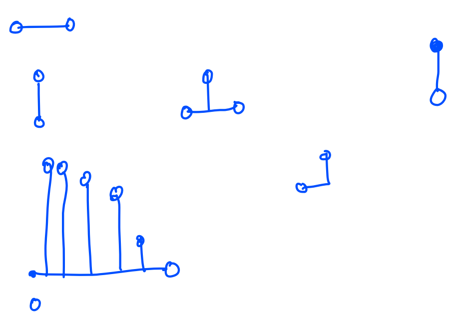
round patch, not including boundary pts.

$Y =$ the infinite comb $\subseteq \mathbb{R}^2$

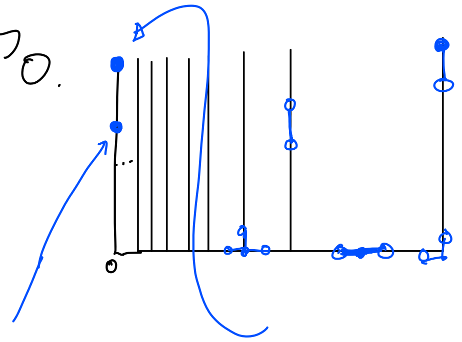


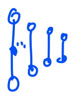
vertical lines at $x = 1/n$.

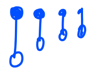
a basis nbhd looks like:

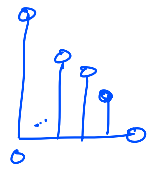


Also including the line at 0.









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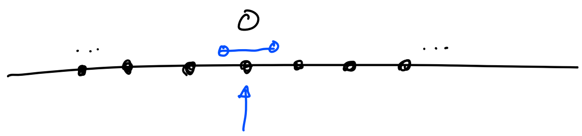
subspaces of $\mathbb{R}$

$\underline{\mathbb{Z} \subseteq \mathbb{R}}$



a small nbhd around 0

looks like: $\{0\}$

in $\mathbb{Z}$, subspace of $\mathbb{R}$,

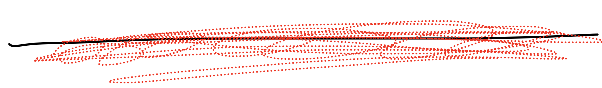
$\{n\}$ is open for any $n \in \mathbb{Z}$.

So in $\mathbb{Z}$, subspace top from $\mathbb{R}$ is the discrete top.

(all sets are open).

The discrete top makes a space of individual isolated points, like $\mathbb{Z} \subseteq \mathbb{R}$.

$\mathbb{Q} \subseteq \mathbb{R}$



a small nbhd looks like $(a,b) \cap \mathbb{Q}$

not discrete.