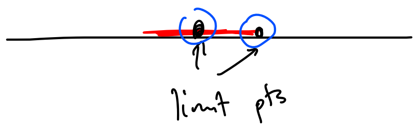
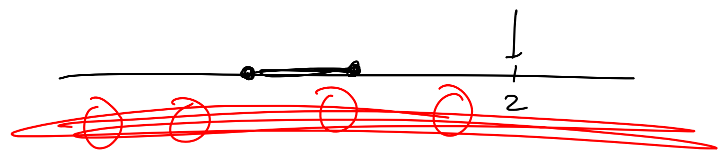
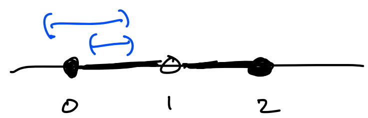
$\frac{1}{n} \to x \quad \forall x \in \mathbb{R}$ with $\mathbb{R}_{fc}$.



$[0,1]$ with fc.



$Y = [0,1) \cup (1,2]$



Y with subspace top from $\mathbb{R}$.

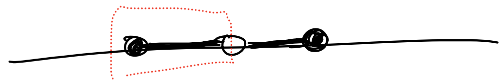
Like $(1/3, 2/3)$ is open in Y

or $[0,1)$ is open in Y

since $[0,1) = \underbrace{(-1,1)}_{\text{open in } \mathbb{R}} \cap Y$

also $(1,2]$ is open in Y,

since $(1,2] = \underline{(1,5)} \cap Y$



In Y, the complement of $[0,1)$ is

$$Y - [0,1) = (1,2]$$

So $[0,1)$ is open, and its complement is open
closed

So $[0,1)$ is closed and open.

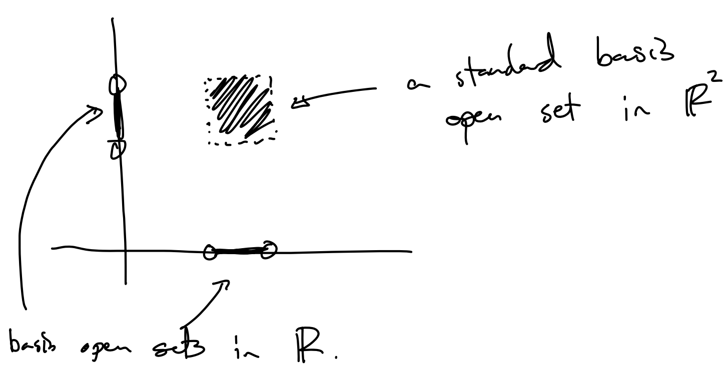
also $(1,2]$ is clopen.

The Product Topology

Given top. spaces X & Y, we can make $X \times Y$ into a top. space.

Like $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

standard top. on $\mathbb{R}^2$ is the product of $(\text{standard } \mathbb{R}) \times (\text{standard } \mathbb{R})$



Def Given top spaces X & Y, let $X \times Y$ be the product, and the <u>product topology</u> is generated by the basis:

$$\mathcal{B} = \left\{ U \times V \;\middle|\; \begin{array}{l} U \text{ is open in } X \\ V \text{ is open in } Y \end{array} \right\}$$

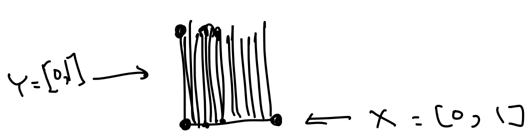
Not all open sets look like rectangles, but these are a basis.

in $\mathbb{R}^2$

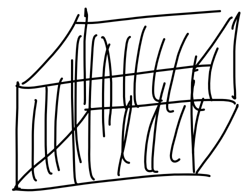
is open, even though it's not equal to $U \times V$.

Visualizing a product: $\underset{X}{[0,1]} \times \underset{Y}{[0,1]}$

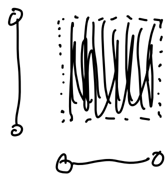
imagine the 2 sets as perpendicular, and each point of X is ~~attached~~ to a copy of Y.



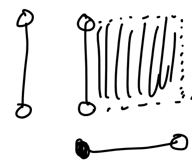
$\underbrace{[0,1] \times [0,1]}_{\text{square}} \times [0,1]$



$(0,1) \times (0,1)$

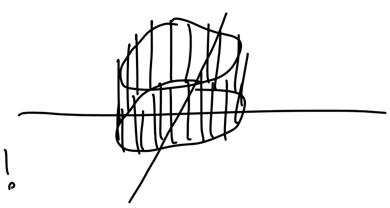


$[0,1) \times (0,1)$



S^1 = unit circle.

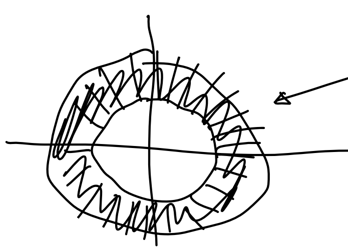
What is $S^1 \times [0,1]$



$\mathbb{R}^2$

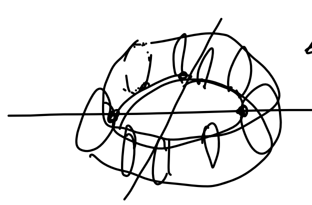
makes a cylinder!

OR: (topologically equiv) $S^1 \times [0,1]$

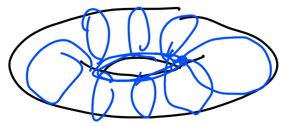


the annulus

$S^1 \times S^1$



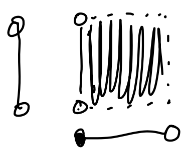
torus



What do opensets look like in:

$$\mathbb{R}_\ell \times \mathbb{R} \quad \text{vs} \quad \mathbb{R}_\ell \times \mathbb{R}_\ell$$

$$[0,1) \times (0,1)$$



a basis open set in $\mathbb{R}_\ell \times \mathbb{R}$

$$\mathbb{R}_\ell \times \mathbb{R}_\ell$$

$$[0,1) \times [0,1)$$