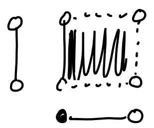
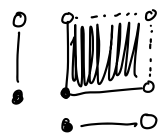
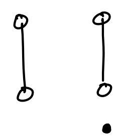
Product Topology

in $X \times Y$, the basis open sets are
$U \times V$ where U is open in X
V is open in Y.

in $\mathbb{R}_\ell \times \mathbb{R}$ $\mathbb{R}_\ell \times \mathbb{R}_\ell$ $\boxed{\mathbb{R}_{disc} \times \mathbb{R}}$ ← "vert. interval top"

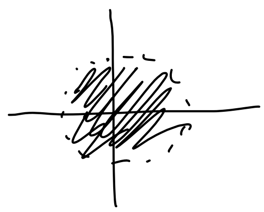




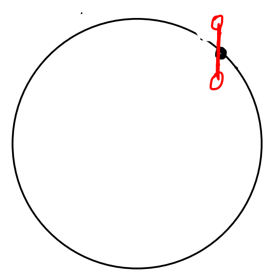


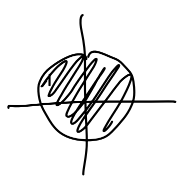
the basis nbhd of $\mathbb{R}_{disc} \times \mathbb{R}$ in $\mathbb{R}^2$

in each topology: which of these is open?

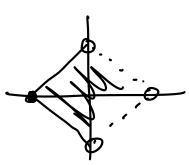


in $\mathbb{R}_\ell \times \mathbb{R}$: open
$\mathbb{R}_\ell \times \mathbb{R}_\ell$: open
$\mathbb{R}_{disc} \times \mathbb{R}$: open

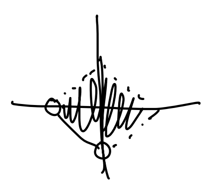




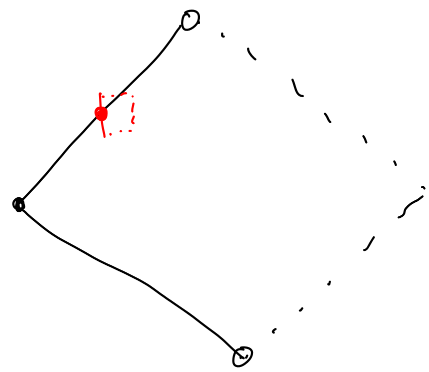
$\mathbb{R}_\ell \times \mathbb{R}$: not open
$\mathbb{R}_\ell \times \mathbb{R}_\ell$: not open
$\mathbb{R}_d \times \mathbb{R}$: not open



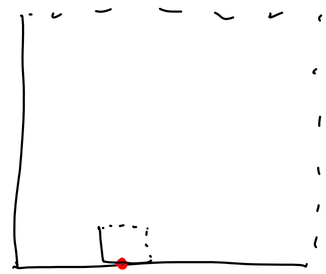
not open



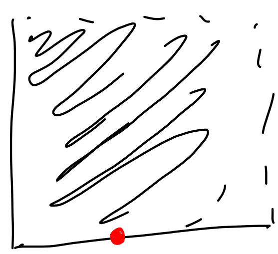
not open



$\mathbb{R}_\ell \times \mathbb{R}$ not open



is open in $\mathbb{R}_\ell \times \mathbb{R}_\ell$

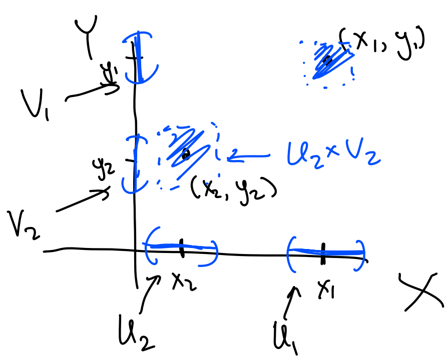


using $\mathbb{R}_\ell \times \mathbb{R}$

You try:

<u>Thm</u> If X & Y are Hausdorff, then $X \times Y$ is <u>Hausdorff.</u>

$\forall a, b \in X$, $\exists$ nbhds U, V with $U \cap V = \emptyset$



<u>Pf</u> Take $(x_1, y_1), (x_2, y_2) \in X \times Y$.
We'll find nbhds of each in $X \times Y$ with no overlap.

Since X is Haus, $\exists$ nbhds U_1 & U_2 in X
with $x_1 \in U_1$, $x_2 \in U_2$, and $U_1 \cap U_2 = \emptyset$.

Similarly $\exists$ nbhds V_1 & V_2 in Y
with $y_1 \in V_1$, $y_2 \in V_2$, $V_1 \cap V_2 = \emptyset$.

Then $U_1 \times V_1$ is a nbhd of (x_1, y_1) in $X \times Y$.

($U_1 \times V_1$ is open since U_1 & V_1 are open, and we're using product top.)

Similarly, $U_2 \times V_2$ is a nbhd of (x_2, y_2) in $X \times Y$.

And $(U_1 \times V_1) \cap (U_2 \times V_2) = \emptyset$

since $U_1 \cap U_2 = \emptyset$ and $V_1 \cap V_2 = \emptyset$.

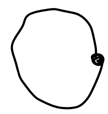
The Quotient Topology

We create a new top. space by considering some pts to be equivalent with one another.

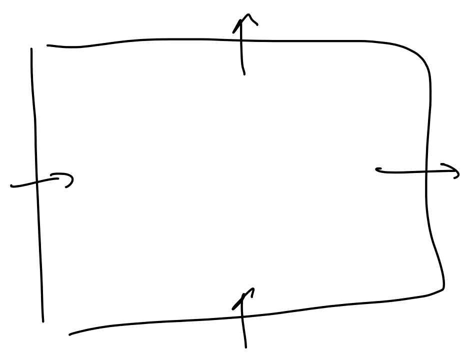
"gluing points together"

$[0,1]$

, glue the 2 endpts together, leave everything else.

we get 

S^1 is a quotient of $[0,1]$, obtained by identifying the endpts.



PacMan lives on a cylinder.