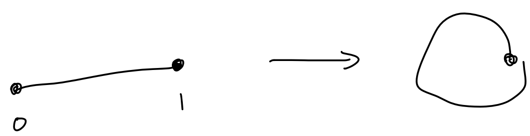
Quotient topology

(using equivalence classes)

X with some equivalence $\sim$,

let X^* be the set of equiv. classes.

The quot. top. makes X^* into a top. space.

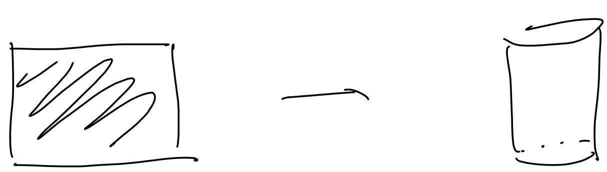


The equivalence classes are: $\{x\}$ for $0 < x < 1$,

and $\{0, 1\}$

These classes make "a partition" of X.

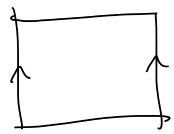
A cylinder from $[0,1] \times [0,1]$



$[0,1] \times [0,1]$

Classes look like: $A_{(x,y)} = \{(x,y)\}$ when $0 < x < 1$ and $0 \leq y \leq 1$

also $B_y = \{(0,y),(1,y)\}$ when $0 \le y \le 1$

 is a cylinder

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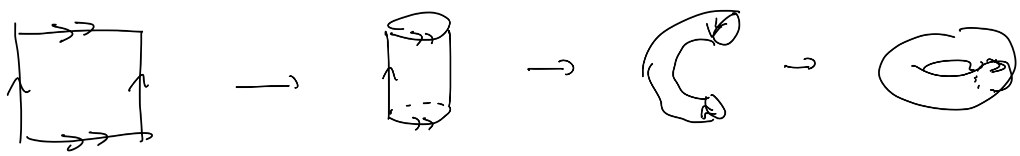
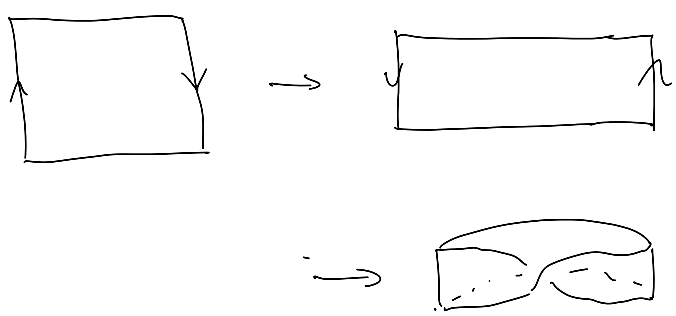


Equiv classes: $A_{(x,y)} = \{(x,y)\}$ when $0 < x < 1$, $0 < y < 1$

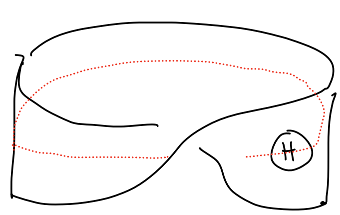
$B_y = \{(0,y),(1,y)\}$ for $0 \le y \le 1$

$C_x = \{(x,0),(x,1)\}$ for $0 \le x \le 1$



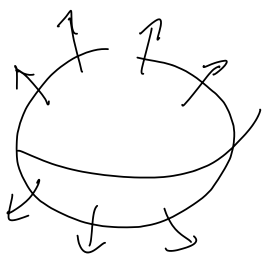
"The Möbius strip"

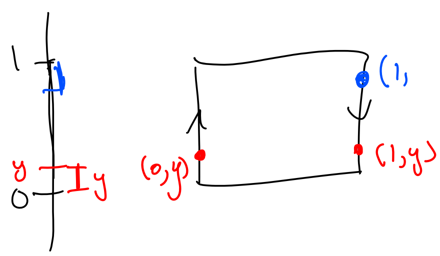
This is a "1-sided" shape



A H can slide around and become a T

The Möbius strip is "non-orientable"

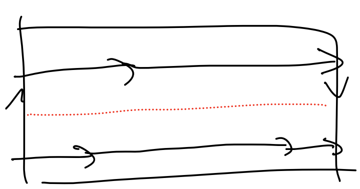


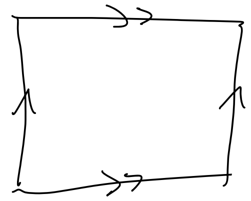


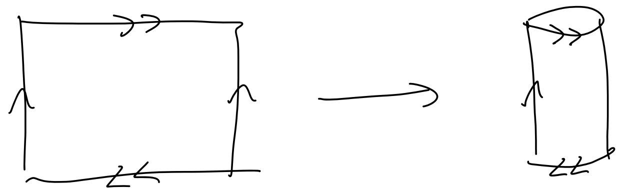
Equivalence classes:

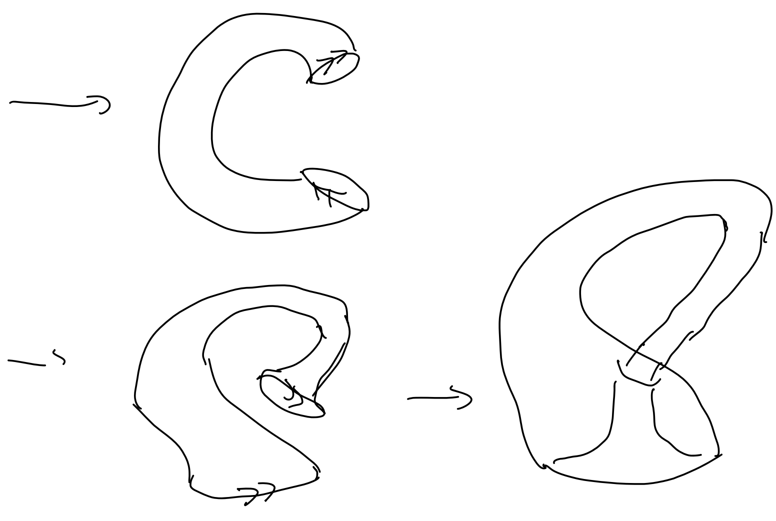
$$A_{(x,y)} = \{(x,y)\} \quad \text{if} \quad \begin{array}{l} 0<x<1 \\ 0 \le y \le 1 \end{array}$$

$$B_y = \{(0,y),(1,1-y)\} \quad \text{if} \quad 0 \le y \le 1$$



 is the torus,



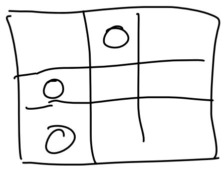


To make this in $\mathbb{R}^3$, we need to allow the shape to pass thru itself.

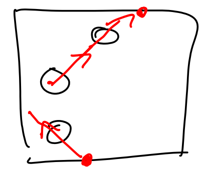
It "really" lives in $\mathbb{R}^4$

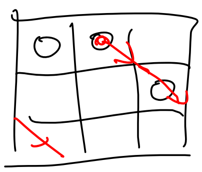
This is the <u>Klein</u> <u>Bottle</u> K.

We could play Tic-Tac-Toe on K

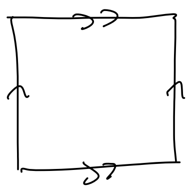


is winning:

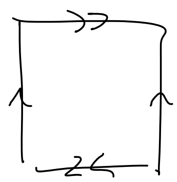




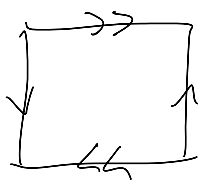
is not winning



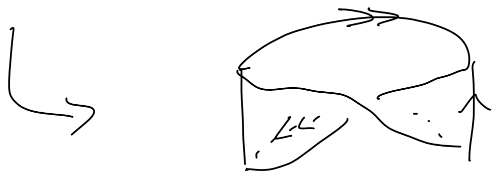
torus



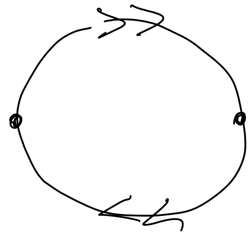
Klein



"the projective plane" $\mathbb{RP}^2$



Same as:



Tic - Tac - Toe