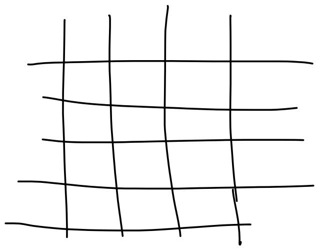
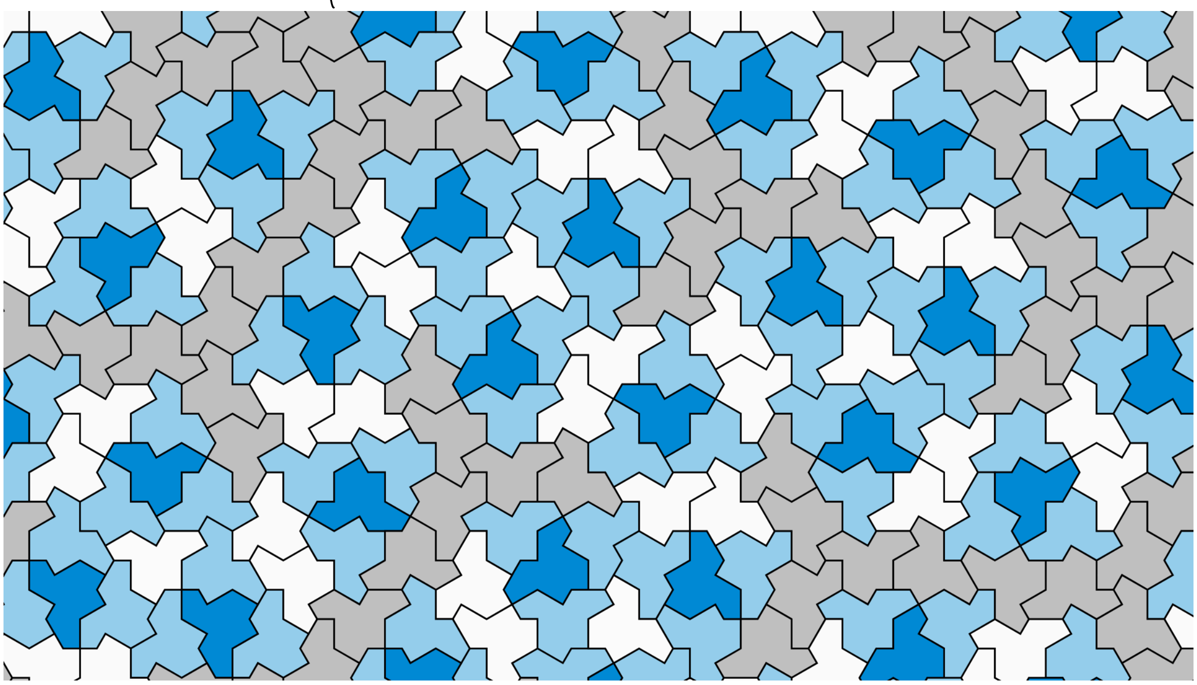
Aperiodic Tiling (Penrose tiling)

2 shapes, no periodicity



periodic



Metric Spaces

(vs Top. spaces)

Metric space $\Rightarrow$ Top space

Can we use the metric to create a topology?

we define $B(x,\varepsilon) = \{ y \mid d(x,y) < \varepsilon \}$,

use the balls as basis nbhds for a topology.

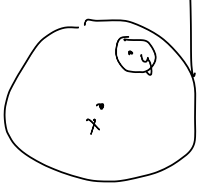
This is called the topology induced by the metric

We need to show that the balls make a basis.

last time

Lemma if $y \in B(x,\varepsilon)$, then $\exists\, \delta > 0$

s.t. $B(y,\delta) \subseteq B(x,\varepsilon)$.

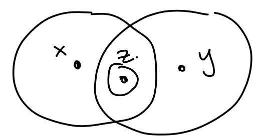


Thm The balls make a basis.

Pf covering (WTS any $x \in X$ is inside a basis set)

Let $x \in X$, then $x \in B(x,1)$.

intersections



let $B(x, \varepsilon_1)$ & $B(y, \varepsilon_2)$ be balls with

$$z \in B(x, \varepsilon_1) \cap B(y, \varepsilon_2)$$

By the lemma, $\exists\, \delta$ so small that

$$B(z, \delta) \subseteq B(x, \varepsilon_1)$$

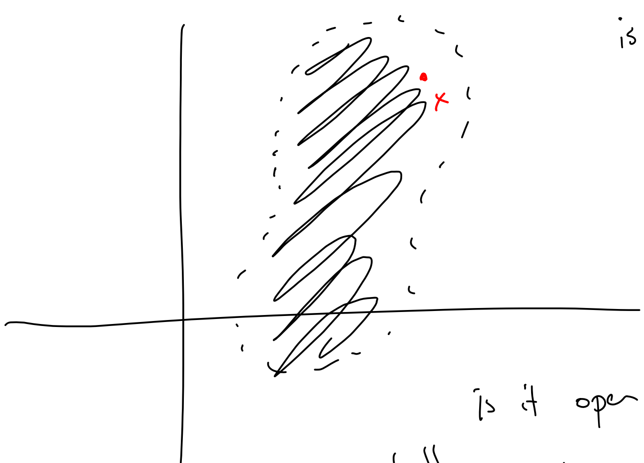
and $B(z, \delta) \subseteq B(y, \varepsilon_2)$

so $B(z, \delta) \subseteq B(x, \varepsilon_1) \cap B(x, \varepsilon_2)$

Shown.

So any metric defines an induced topology.

in $\mathbb{R}^2$, we had d, d_T, d_M, d_{disc}.



is it open using d ← standard euclidean metric?

Yes, since any pt in there has a standard ball contained in the set.

is it open using d_T? Yes!

d_T-ball →

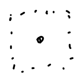
we can also put d_T-balls around any point.

The topologies induced by standard metric d

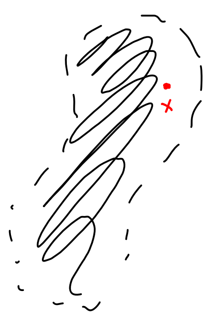
vs the metric d_T

are <u>the same</u>!

Balls look different, but they create the same open sets.

d_M ?  This again is the same topology.

d_{disc} ? balls look like indiv. points.



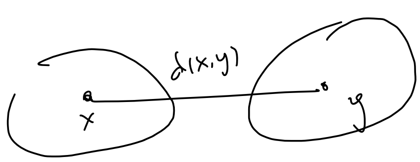
This is open using d_{disc} - balls.

Also 

is open since $\{x\}$ is a ball.

Actually <u>all</u> sets are open using d_{disc}

So d_{disc} induces the discrete topology

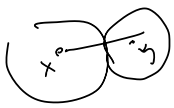


Thm Any metric space is Hausdorff.

Pf Take $x, y \in X$, we'll find U, V open with $x \in U$, $y \in V$, $U \cap V = \emptyset$.

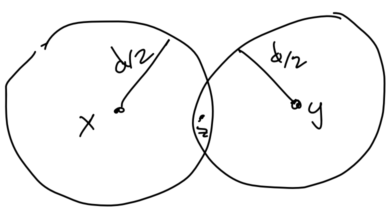
Let $U = B(x, \frac{d(x,y)}{2})$

$V = B(y, \frac{d(x,y)}{2})$



So $x \in U$ & $y \in V$.

$U \cap V = \emptyset$: To get a contradiction, assume $z \in U \cap V$



By Δ-ineq,

$$d(x,y) \le d(x,z) + d(z,y) < \frac{d(x,y)}{2} + \frac{d(x,y)}{2} = d(x,y)$$

So $d(x,y) < d(x,y)$ Contradiction

Metric spaces are topologically "nice"

Hausdorff, and other nice properties.

Any metric induces a topology.

Can we do the other way? NO

All metric spaces are Hausdorff,

so e.g. $\mathbb{R}_{fc}$ is not a metric space.

"$\mathbb{R}_{fc}$ is not metrizable"

the trivial top is not metrizable.