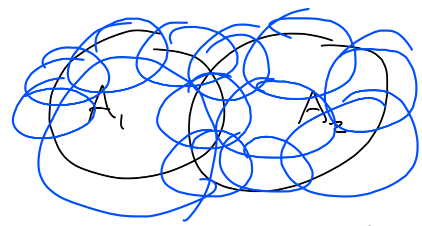
Compact sets (in $\mathbb{R}^n$) are closed & bounded.

If A_1 & A_2 are compact

is $A_1 \cup A_2$ compact? Yes

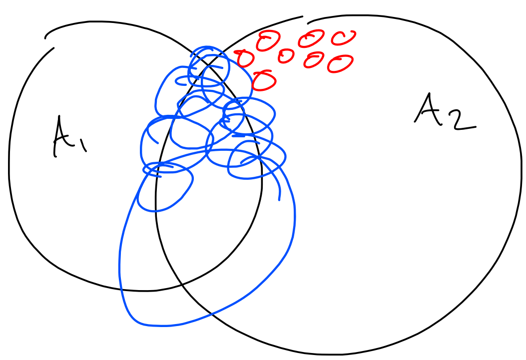


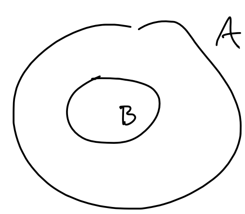
$A_1 \cap A_2$? Yes ← when X is Hausdorff

$A_1 \times A_2$? Yes

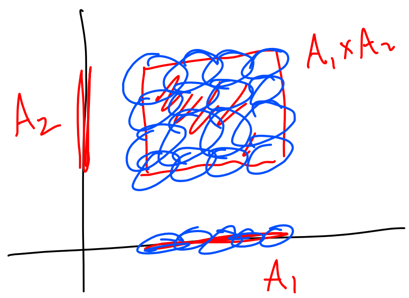
if A is compact & $B \subset A$, ← let $A = [-1, 1]$,

is B compact? $\boxed{\text{NO}}$ $B = (-1, 1)$





Thm If A_1 & A_2 are compact, then $A_1 \times A_2$ is compact.



Fake Proof Let $\mathcal{O}$ be an open cover of $A_1 \times A_2$.

Assume each set in the cover looks like $U \times V$,

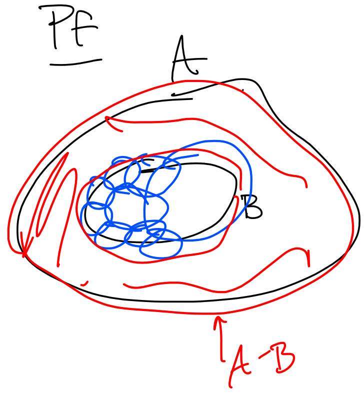
Then the U's make a cover of A_1, so only finitely many U's are needed.

Also the V's make a cover of A_2, so only finitely many V's are needed.

So we only need finitely many sets like $U \times V$ to cover $A_1 \times A_2$.

Thm If A is compact & $B \subseteq A$ & B is closed, then B is compact.

Pf



Let $\mathcal{O}$ be an open cover of B.

[Let's add stuff into the cover to make it cover all of A; it's missing $A-B$]

Since B is closed, $A-B$ is open,

So $\mathcal{O} \cup \{A - B\}$ is an open cover of A.

So we need only fin. many sets from $\mathcal{O} \cup \{A - B\}$ to cover A,

So we only need fin. many sets from $\mathcal{O}$ to cover B.