

Name: _____

Math 1015: Exam #1

Question 1. Please find the winner using plurality. Show enough work so that I can tell what you're doing.

4	2	3
A	B	C
B	C	B
C	A	A

plurality: A: 4
 B: 2 A wins!
 C: 3

Question 2. Please find the winner using ranked choice voting. Show enough work so that I can tell what you're doing.

4	2	3	1
A	B	D	C
B	C	B	A
C	A	A	D
D	D	C	B

Round 1
A: 4
B: 2
~~C: 1~~
D: 3

Round 2

<u>4</u>	<u>2</u>	<u>3</u>	<u>1</u>	A: 5
A	B	D	A	B: 2
B	A	B	D	D: 3
D	D	A	B	

Round 3

<u>4</u>	<u>2</u>	<u>3</u>	<u>1</u>	A: 7
A	A	D	A	
D	D	A	D	D: 3

A wins

Question 3. Please find the winner using Condorcet's method, or say that there is no winner.

	4	2	3
A	B	C	
B	C	B	
C	A	A	

$$A \checkmark \textcircled{B}: \quad A: 4 \\ B: 2+3=5$$

$$A \checkmark \textcircled{C}: \quad A: 4 \\ C: 2+3=5$$

$$\textcircled{B} \checkmark C: \quad B: 4+2=6 \\ C: 3$$

B wins!

Question 4. Please find the winner using the Borda count.

	4	2	3
2 A	B	C	
1 B	C	B	
0 C	A	A	

$$A: 4 \times 2 + 2 \times 0 + 3 \times 0 = 8$$

$$B: 4 \times 1 + 2 \times 2 + 3 \times 1 \\ 4 + 4 + 3 = \textcircled{11}$$

$$C: 4 \times 0 + 2 \times 1 + 3 \times 2 \\ 2 + 6 = 8$$

B wins!

Question 5. Please explain why ranked choice voting satisfies the majority criterion.

Imagine a majority of voters rank X in 1st place.
Then X will always have the most # of 1st place rankings in each RCV round, so X will never be eliminated.
Thus X wins the election using RCV.

Question 6. Please explain why Condorcet's method satisfies the unanimity criterion.

Imagine all voters rank X above Y .
Then X will beat Y in a 1 vs 1 matchup,
so Y will not be the winner using
Condorcet's method.

Question 7. Please use this example to explain why the Borda count does not satisfy the Condorcet Winner Criterion.

	3	4	
2	A	B	
1	C	A	
0	B	C	

Borda winner:

$$A: 3 \times 2 + 4 \times 1 = 10$$

$$B: 3 \times 0 + 4 \times 2 = 8$$

$$C: 3 \times 1 + 4 \times 0 = 3$$

A wins with Borda.

Condorcet winner:

A vs B: A: 3
 B: 4

B is the Condorcet winner,

A vs C: A: 3+4
 C:

but B loses with Borda.

B vs C: B: 4
 C: 3

So Borda does not satisfy CWK.

Question 8. Please explain why the plurality system satisfies monotonicity.

Imagine X wins with plurality, and then we boost X. Then X's amount of 1st-place rankings may increase, but nobody else's will increase, so X will still be the winner using plurality.

Question 9. Please use the following example to demonstrate that plurality does not satisfy IIA. (You don't need to write a proof.)

4	3	2
ⓑ	A	C
C	ⓑ	A
A	C	ⓑ

plurality: A: 3 B wins!
 B: 4
 C: 2

But if $\begin{matrix} C \\ A \\ \textcircled{B} \end{matrix}$ changes to $\begin{matrix} A \\ C \\ \textcircled{B} \end{matrix}$, (this doesn't move anyone past the winner) then we get:

4	3	2	
ⓑ	A	A	A: 5
C	B	C	B: 4
A	C	B	C: 0

A wins!

Question 10. Use this sample election to show how some of the voters can manipulate the election if we're using Borda. Write some words explaining why your example qualifies as a manipulation.

6	3	2
ⓐ	C	B
C	ⓐ	C
B	B	ⓐ

OG result: A: $12 + 3 + 0 = 15$
 B: $0 + 0 + 4 = 4$ A wins
 C: $6 + 6 + 2 = 14$

The $\begin{matrix} B \\ C \\ A \end{matrix}$ voters get their worst outcomes so they will try to manip. by changing $\begin{matrix} B \\ C \\ A \end{matrix} \rightarrow \begin{matrix} C \\ B \\ A \end{matrix}$

Then we get:

6	3	2
2	A	C
1	C	A
0	B	B
		A

A: 15
 B: 2
 C: $6 + 6 + 4 = \textcircled{16}$

So C wins, which is a preferable outcome to the $\begin{matrix} B \\ C \\ A \end{matrix}$ voters.