

Name: _____

Math 1015: Exam #1

Question 1. Please find the winner using plurality. Show enough work so that I can tell what you're doing.

4	2	3
A	B	C
B	C	B
C	A	A

plurality: A: 4
 B: 2 A wins!
 C: 3

Question 2. Please find the winner using ranked choice voting. Show enough work so that I can tell what you're doing.

4	2	3	1
A	B	D	C
B	C	B	A
C	A	A	D
D	D	C	B

Round 1
A: 4
B: 2
~~C: 1~~
D: 3

Round 2

4	2	3	1	A: 5
A	B	D	A	B: 2
B	A	B	D	D: 3
D	D	A	B	

Round 3

4	2	3	1	A: 7
A	A	D	A	
D	D	A	D	D: 3

A wins

Question 3. Please find the winner using Condorcet's method, or say that there is no winner.

	4	2	3
A	B	C	
B	C	B	
C	A	A	

A vs \textcircled{B} :
 A: 4
 B: $2+3=5$

A vs \textcircled{C} :
 A: 4
 C: $2+3=5$

\textcircled{B} vs C:
 B: $4+2=6$
 C: 3

\textcircled{B} wins!

Question 4. Please find the winner using the Borda count.

	4	2	3
2 A	B	C	
1 B	C	B	
0 C	A	A	

A: $4 \times 2 + 2 \times 0 + 3 \times 0 = 8$

B: $4 \times 1 + 2 \times 2 + 3 \times 1$
 $4 + 4 + 3 = \textcircled{11}$

\textcircled{B} wins!

C: $4 \times 0 + 2 \times 1 + 3 \times 2$
 $2 + 6 = 8$

Question 5. Please prove that Condorcet's method satisfies the majority criterion.

Imagine a majority of voters rank X 1st.
Then X must win all 1-on-1 matchups,
so X wins using Condorcet's method.

Question 6. Please prove that ranked choice voting satisfies the unanimity criterion.

Imagine all voters rank X above Y .
Then Y will be eliminated before X ,
so Y will not win the election.

Another correct proof:

Imagine all voters rank X above Y .
Then Y gets no 1st-place rankings, so Y is eliminated in round 1.
So Y will not win the election.

Question 7. Please use this example to explain why ranked choice voting does not satisfy the Condorcet Winner Criterion.

3	4	2
A	B	C
C	C	A
B	A	B

We need to show the Cond. winner is different from the RCV winner.

Condorcet.

A vs B: A: 3 + 2
B: 4

A vs C: A: 3
C: 4 + 2

B vs C: B: 4
C: 3 + 2

C is the Cond. winner

RCV

Round 1:

A: 3			
B: 4			
C: 2			

Rd 2

3	4	2	A: 5
A	B	A	B: 4
B	A	B	

A is the RCV winner

Question 8. Please prove that the Borda count satisfies monotonicity.

Imagine X wins with Borda, then we boost X. After the boost, X will get more points (and nobody else will get more points) so X is still the winner.

Question 9. Please use the following example to demonstrate that plurality does not satisfy IIA. (You don't need to write a proof.)

4	3	2
ⓑ	A	C
C	ⓑ	A
A	C	ⓑ

OG plurality winner: A: 3
B: 4
C: 2

ⓑ

but I can change $\begin{matrix} C \\ A \\ \textcircled{B} \end{matrix} \rightarrow \begin{matrix} A \\ C \\ \textcircled{B} \end{matrix}$ without moving anyone past the winner,

this changes the result:

4	3	2	
B	A	A	A: 5
C	B	C	B: 4
A	C	B	C: 0

ⓐ

Question 10. Use this sample election to show how some of the voters can manipulate the election if we're using plurality. Write some words explaining why your example qualifies as a manipulation.

4	3	2
ⓐ	C	B
C	ⓐ	C
B	B	ⓐ

OG result: A: 4
B: 2
C: 3

ⓐ

The $\begin{matrix} B \\ C \\ A \end{matrix}$ voters got their worst possible result, so will try to improve them by changing $\begin{matrix} B \\ C \\ \textcircled{A} \end{matrix} \rightarrow \begin{matrix} C \\ B \\ \textcircled{A} \end{matrix}$. Now it says:

4	3	2	
A	C	C	A: 4
C	A	B	B: 0
B	B	A	C: 5

Now C wins, which is an improvement for the $\begin{matrix} B \\ C \\ A \end{matrix}$ voters.