

Name: _____

Math 1015: Exam #2

Question 11. Somebody says: "the random dictator method is stupid because it basically ignores the votes and just chooses the winner at random." Do you agree or disagree? Write some words explaining why.

I disagree! Random dictator does not "ignore the votes", it chooses one vote at random. This method does perfectly reflect the will of the voters, but only probabilistically.

Question 12. Please translate this ranked election into an approval voting election, assuming that each voter approves of their top 3 choices. Show the chart of approval ballots that would result, and determine the winner using approval voting.

3	4	2	1
A	B	C	A
D	C	A	B
C	A	D	D
B	D	B	C

	3	4	2	1	
A	✓	✓	✓	✓	A: 10
B		✓		✓	B: 5
C	✓	✓	✓		C: 9
D	✓		✓	✓	D: 6

A wins!

Question 13. Please consider the weighted voting system: $[20 : 12, 5, 4, 2, 2, 1]$

a) Identify any dictators, or say that there are none.

No dictators, since no voter is 15 all by themselves.

b) Identify any voters with veto power, or say that there are none.

12 has veto power, since without 12 it's $5+4+2+2+1=14$ so 12 is necessary to meet the quota.

5 does not, since without 5 it's $12+4+2+2+1=21$

c) Identify any dummies, or say that there are none.

No dummies: even the 1 can matter sometimes.

like in $12+5+2+1=20$

Question 14. Please find the Shapley-Shubik power index for the weighted system $[20 : 11, 10, 9]$

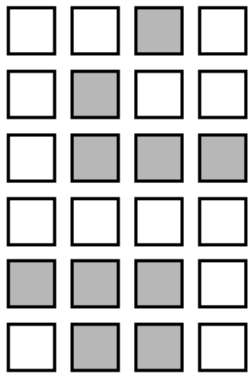
A B C

<u>perms</u>	<u>weights</u>	<u>pivotal</u>	
A B C	11 <u>10</u> 9	B	A: $\frac{4}{6}$ B: $\frac{1}{6}$ C: $\frac{1}{6}$
A C B	11 <u>9</u> 10	C	
B A C	10 <u>11</u> 9	A	
B C A	10 9 <u>11</u>	A	
C A B	9 <u>11</u> 10	A	
C B A	9 10 <u>11</u>	A	

Question 15. Please find the Banzhaf power index for the weighted system [20 : 14, 12, 6]

			total	A	B	C	
A	B	C	32	✓			A: 3/5
A	B		26	✓	✓		B: 1/5
A		C	20	✓		✓	C: 1/5
<hr/>							
	B	C	18				
A			14				
	B		12				
		C	6				
		∅	6				

Question 16. Here is a map with 24 voters who are members of 2 political parties (the Dark and the Light). We want to divide them into 3 districts of 8 voters each. Please determine all the possible outcomes for how many districts can be won by each party.



Threshold to win a district is 5.

D has 10, so they could win up to 2 districts, since $5 \times 2 = 10$.

L has 14, so they could also win up to 2.

(L can't win 3, since this would require $5 \times 3 = 15$)

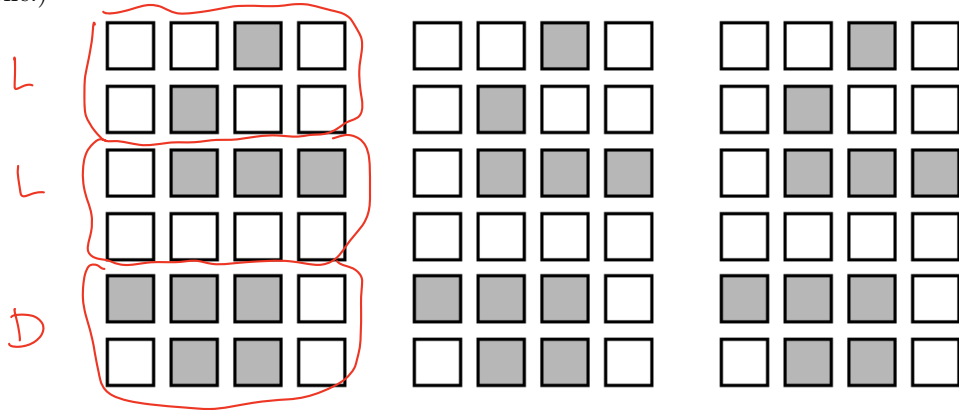
~~3D 0L~~
 2D 1L
 1D 2L
~~0D 3L~~

So the only possible outcomes are

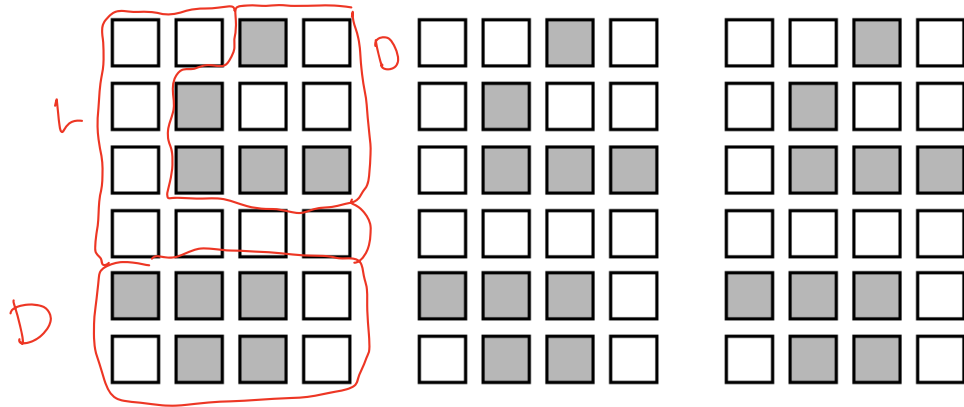
2D 1L

1D 2L

Question 17. a) Please draw 3 districts of 8 voters each so that the light party wins 2 districts, and the dark wins 1. (I am giving you 3 pictures in case you mess up. If you do it right, you will only need to use one.)



b) Please draw 3 districts of 8 voters each so that the light party wins 1 district, and the dark wins 2. (I am giving you 3 pictures in case you mess up. If you do it right, you will only need to use one.)



Exam 2 old ones:

#1

3	2	4	2
A	B	C	C
B	A	B	A
C	C	A	B

Plurality:

A: 3

B: 2

C: 6

C wins!

#2

3	2	4	2
A	B	C	C
B	A	B	A
C	C	A	B

Round 1:

A: 3

B: 2

C: 6

B is eliminated.

Round 2

3	2	4	2
A	A	C	C
C	C	A	A

A: 5

C: 6

C wins!

#3

3	2	4	2
A	B	C	C
B	A	B	A
C	C	A	B

A v B

A: 3+2=5

B: 2+4=6

A v C

A: 3+2=5

C: 4+2=6

B v C

B: 3+2=5

C: 4+2=6

C is the Condorcet winner

#4

3	2	4	2
A	B	C	C
B	A	B	A
C	C	A	B

A: $3 \times 2 + 2 \times 1 + 2 \times 1 = 10$

B: $3 \times 1 + 2 \times 2 + 4 \times 1 = 11$

C: $4 \times 2 + 2 \times 2 = 12$

C wins!

5. Proof Imagine X is ranked 1st by a majority of voters. Then X gets the most 1st-place rankings, so X wins using plurality.

#6 Proof Imagine X is ranked above Y on all ballots. Then X gets more points than Y, so Y does not win using Borda.

#7

3	4	2	
A	B	C	
C	C	A	
B	A	B	

 Here, B is the plurality winner. But the Condorcet winner is: C

<u>A & B</u>	<u>A & C</u>	<u>B & C</u>
A: 3+2=5	A: 5	B: 4
B: 4	C: 4+2=6	C: 3+2=5

Since the Condorcet winner didn't win the election, plurality does not satisfy CWC.

#8 Proof Imagine X is the winner, and then we boost X on some ballots. Since X was the dictator's choice originally, they will still be the dictator's favorite after the boost. So X still wins using dictatorship.

#9

Proof Imagine X is the Condorcet winner,
 and then we change ballots without moving anyone
 past X. Then X still will win every
 1v1 matchup, so X still wins the election
 with Condorcet's method.

#10

3	4	2
A	B	C
C	C	A
B	A	B

OB winner:
 Round 1: A:3
 B:4
~~C:2~~

Round 2: A:5
 B:4
A wins

B
 C
 A voters can instead vote $\begin{matrix} C \\ B \\ A \end{matrix}$. Then it looks like:

3	4	2	Rd 1	A:3	Rd 2	A:3	C wins.
A	C	C		B:4		C:4	
C	B	A		C:4			
B	A	B					

This is a manipulation because it is a preferable
 result for the ones who changed their vote.