# Math 1015: Exam #2

**Question 11.** Consider this election using the random dictator method. Please give the probability for each candidate to win. (Write your probabilities as fractions– you don't need to convert to percentages.)

| 3 | 2 | 4 | 2 |
|---|---|---|---|
| A | B | C | C |
| B | A | B | A |
| C | C | A | B |

⟹ 11 total voters

A: 3/11

B: 2/11

C: 6/11

**Question 12.** Please translate this ranked election into an approval voting election, assuming that each voter approves of their top 2 choices. Show the chart of approval ballots that would result, and determine the winner using approval voting.

| 3 | 4 | 2 |
|---|---|---|
| A | B | C |
| D | C | A |
| C | A | D |
| B | D | B |

|   | 3 | 4 | 2 |
|---|---|---|---|
| A | ✓ |   | ✓ |
| B |   | ✓ |   |
| C |   | ✓ | ✓ |
| D | ✓ |   |   |

A: 5
B: 4
C: 6
D: 3

C wins!

**Question 13.** Please consider the weighted voting system: $[15 : 7, 6, 3, 2, 2, 1]$

a) Identify any dictators, or say that there are none.

No dictators, since no voter is 15 all by themself.

b) Identify any voters with veto power, or say that there are none.

7 has veto power, since without 7 we only get $6+3+2+2+1 = 14$

6 does not, since $7+3+2+2+1 = 15$ so we can reach the quota without 6.

c) Identify any dummies, or say that there are none.

No dummies – even the 1 can matter, like in

$$7+3+2+2+1 = 15$$

**Question 14.** Please find the Shapley-Shubik power index for the weighted system $[20 : 14, 12, 6]$

| perms | weights | | | |
|-------|---------|---|---|---|
| A B C | 14 12̲ 6 | | B | |
| A C B | 14 6̲ 12 | | C | |
| B A C | 12 6 14̲ | | A | |
| B C A | 12 14̲ 6 | | A | |
| C A B | 6 14̲ 12 | | A | |
| C B A | 6 12 14̲ | | A | |

A: 4/6

B: 1/6

C: 1/6

**Question 15.** Please find the Banzhaf power index for the weighted system $[20 : 11, 10, 9]$

|          | total | A | B | C |
|----------|-------|---|---|---|
| A  B  C  | 30    | ✓ |   |   |
| A  B     | 21    | ✓ | ✓ |   |
| A     C  | 20    | ✓ |   | ✓ |
|    B  C  | 19    |   |   |   |
| A        | 11    |   |   |   |
|    B     | 10    |   |   |   |
|       C  | 9     |   |   |   |
| ∅        | 0     |   |   |   |

A: $3/5$

B: $1/5$

C: $1/5$

**Question 16.** Here is a map with 30 voters who are members of 2 political parties: 18 of them are stars, and 12 of them are squares. We want to divide them into 3 districts of 10 voters each. Please determine all the possible outcomes for how many districts can be won by each party.
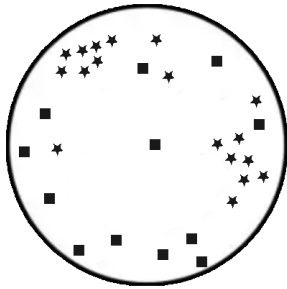
10 voters per district, so the threshold to win a district is 6 votes.

★ have 18, so they can win up to 3 districts, since $6 \times 3 = 18$.
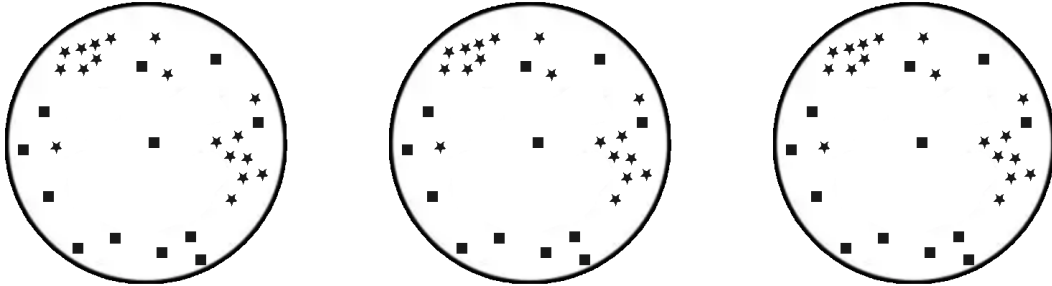
□ have 12, so they can only win up to 2.

So 0★ / 3□ is impossible, but

these others are possible:

3★  0□

2★  1□

1★  2□

3 ★    0 □
2 ★    1 □
1 ★    2 □
~~0 ★    3 □~~

**Question 17.** a) Please draw 3 districts of 10 voters each so that the star party wins 2 districts, and the squares win 1. (I am giving you 3 pictures in case you mess up. If you do it right, you will only need to use one.)



b) Please draw 3 districts of 10 voters each so that the star party wins all 3 districts. (I am giving you 3 pictures in case you mess up. If you do it right, you will only need to use one.)
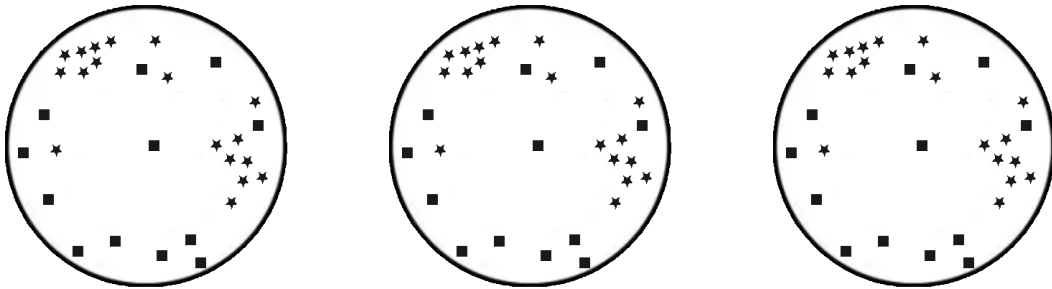
Exam 2 old ones:

**#1**

| 3 | 2 | 4 | 2 |
|---|---|---|---|
| A | B | C | C |
| B | A | B | A |
| C | C | A | B |

Plurality:

A: 3

B: 2

C: 6

C wins!

**#2**

| 3 | 2 | 4 | 2 |
|---|---|---|---|
| A | B | C | C |
| B | A | B | A |
| C | C | A | B |

Round 1:

A: 3

B: 2      B is eliminated.

C: 6

Round 2

| 3 | 2 | 4 | 2 |
|---|---|---|---|
| A | A | C | C |
| C | C | A | A |

A: 5

C: 6        C wins!

**#3**

| 3 | 2 | 4 | 2 |
|---|---|---|---|
| A | B | C | C |
| B | A | B | A |
| C | C | A | B |

A v B

A: 3 + 2 = 5

B: 2 + 4 = 6

A v C

A: 3 + 2 = 5

C: 4 + 2 = 6

B v C

B: 3 + 2 = 5

C: 4 + 2 = 6

C is the Condorcet winner

**#4**

| 3 | 2 | 4 | 2 | |
|---|---|---|---|---|
| A | B | C | C | 2 |
| B | A | B | A | 1 |
| C | C | A | B | 0 |

A: 3×2 + 2×1 + 2×1 = 10

B: 3×1 + 2×2 + 4×1 = 11

C: 4×2 + 2×2 = 12

C wins!

**#5**

| | 4 | 4 | 1 |
|---|---|---|---|
| 2 | B | C | C |
| 1 | A | B | A |
| 0 | C | A | B |

Here, 5 voters rank C 1st, which is a majority of the voters. But C doesn't win using Borda:

A: $4 \times 1 + 1 \times 1 = 5$

B: $4 \times 2 + 4 \times 1 = 12$

C: $4 \times 2 + 1 \times 2 = 10$

B wins!

**#6** <u>Proof</u> Imagine X is ranked above Y on all ballots. Then X has more 1st place rankings than Y, and in fact Y has no 1st place rankings. So Y loses with plurality.

**#8** <u>Proof</u> Imagine X is the Condorcet winner, and then we boost X on some ballots. Then X will still win all 1v1 matchups, so X is still the Condorcet winner.

**#9**

| | 4 | 4 | 1 | |
|---|---|---|---|---|
| | (B) | C | C | 2 |
| | A | (B) | A | 1 |
| | C | A | (B) | 0 |

OG winner:

A: $4 + 1 = 5$

B: $8 + 4 = 12$

C: $8 + 2 = 10$

B wins!

But if we swap A&C here

then C gets 4 more points & A loses 4 pts, so it'll be

A: 1

B: 12

C: 14

We didn't move anyone past B, but it changed the winner. So IIA is violated.

## #10

| 3 | 4 | 2 |
|---|---|---|
| A | (B) | C |
| C | C | A |
| (B) | A | (B) |

OG winner is B

But the C / A / (B) voters can make their 2nd choice win by instead voting A / C / B. This would be an improved result for them.