

Name: \_\_\_\_\_

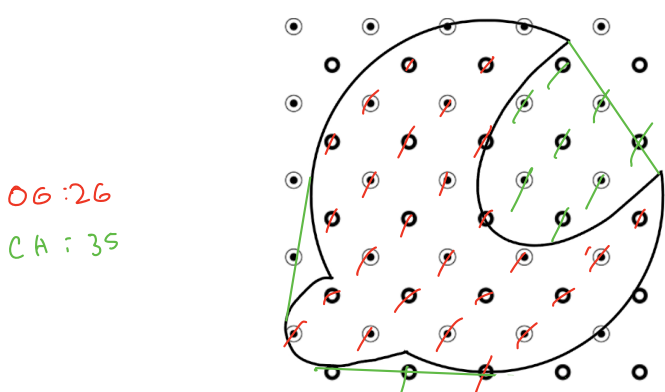
### Math 1015: Exam #3

**Question 18.** Please find the efficiency gap for these votes between the D's and the R's. Your answer should say, "The EG is ??? in favor of ???" You can use an unsimplified fraction in your answer.

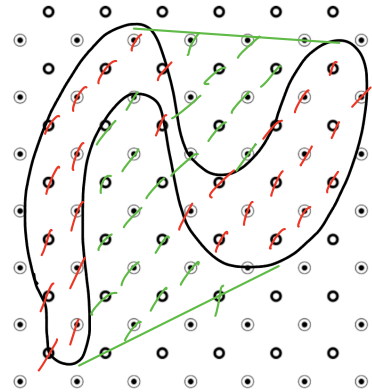
district	D votes	R votes	total	majority	D wasted	R wasted
1	16	14	30	16	0	14
2	25	6	31	16	9	6
3	12	20	32	17	12	3
4	9	22	31	16	9	6
			124		30	29

EG is  $\frac{30 - 29}{124} = \frac{1}{124}$  in favor of R.

**Question 19.** Please find the convex hull ratio of each shape. You can leave your answer as an unsimplified fraction.

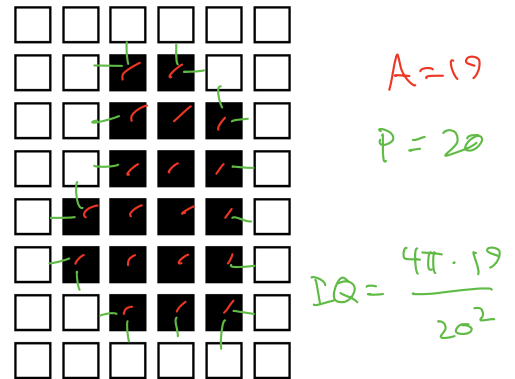
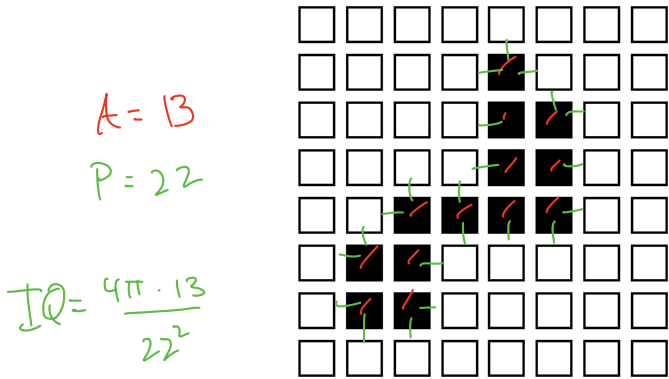


CHR =  $\frac{26}{35}$

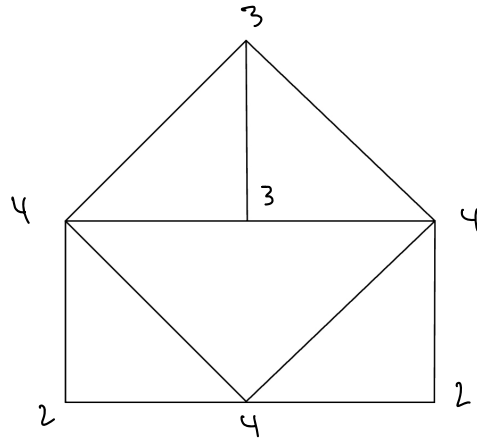


CHR =  $\frac{28}{51}$

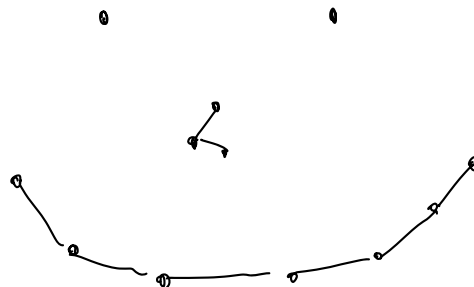
**Question 20.** Please find the isoperimetric quotient of each shape. You can leave your answer as an unsimplified fraction.



**Question 21.**



- Next to each vertex in the graph above, please write its degree. (Every intersection is a vertex.)
- Please draw your own example of a disconnected graph.



**Question 22.** Imagine a graph where each vertex is a different person on campus right now, and two people are connected by an edge if they are both wearing the same color shirt today. (If you're not sure what color your shirt is, or if you're not really wearing a shirt, just make something up.)

- a) What would a typical degree in this graph be? Something like 5? 50? 500? 5,000? 5,000,000? say a few words to explain.

Your degree would be how many other people on campus are wearing the same color shirt as you. 500 seems most reasonable, given those options.

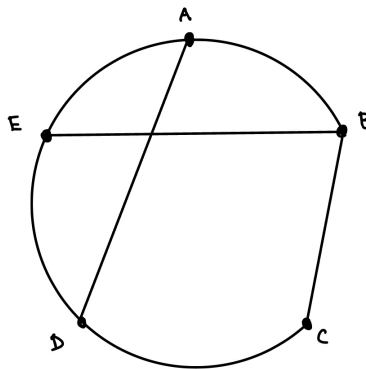
- b) Is there an edge in the graph from you to me (Prof Staecker, who is wearing a blue shirt)? Say a few words to explain why or why not.

If you're wearing a blue shirt, there is.  
otherwise no.

- c) Is this graph connected? Say a few words to explain why or why not.

No - each person can only be connected to others having the same color shirt.

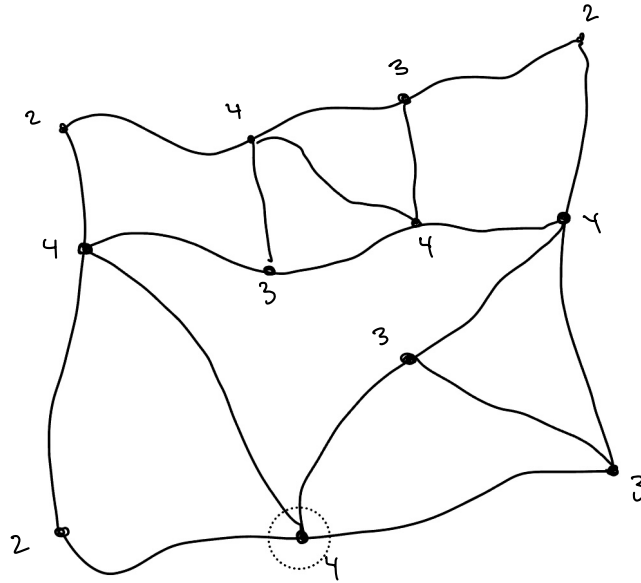
**Question 23.** Please write the formal description of this graph:



vertices:  $\{A, B, C, D, E\}$

edges:  $\{(A, B), (A, D), (A, E), (B, C), (B, E), (C, D), (D, E)\}$

Question 24.



This graph represents a network of roads and intersections. I am planning the Pretty Pony Parade, and I want my parade route to cover every street once without repeating any streets.

- a) Is it possible to do this if the parade starts and ends at the circled intersection? Explain why or why not in a few words.

No - there are some odd degrees, so we can't do an Euler circuit.

- b) Is it possible to do this if the parade starts and ends at different places (not necessarily the circled intersection)? Explain why or why not in a few words.

No! There are more than 2 odd vertices, so we can't do an Euler path.

# Old ones answers

Question 1. Please find the results of this election using plurality:

4	3	4	1	3	2
A	B	B	C		
B	C	A	A		
C	A	C	B		

A: 4

B: 5

C: 2

B wins.

Question 2. Use the election from Question 1 and find the results using ranked choice voting.

Round 1:

A: 4

B: 5

~~C: 2~~

Rd 2:

4	4	1	2
A	B	B	A
B	A	A	B

A: 6

B: 5

A wins

Question 3. Use the election from Question 1 and find the results using Condorcet's method.

A vs B

A: 4 + 2

B: 4 + 1

A vs C

A: 3 + 1

C: 4 + 2

B vs C

B: 4 + 4 + 1

C: 2

No Condorcet winner.

Question 4. Use the election from Question 1 and find the results using Borda count.

	4	4	1	2
2	A	B	B	C
1	B	C	A	A
0	C	A	C	B

A:  $4 \cdot 2 + 4 \cdot 0 + 1 \cdot 1 + 2 \cdot 1$

$8 + 1 + 2 = 11$

B:  $4 \cdot 1 + 4 \cdot 2 + 1 \cdot 2$

$4 + 8 + 2 = 14$

C:  $4 \cdot 0 + 4 \cdot 1 + 1 \cdot 0 + 2 \cdot 2$

$4 + 4 = 8$

B wins

Question 5. Please prove that Condorcet's method satisfies the majority criterion.

Imagine a majority of voters rank X first.

Then X automatically wins any pair comparison,

so X wins using Condorcet's method.

**Question 6.** Please prove that Borda count satisfies the unanimity criterion.

Imagine all voters rank  $X$  above  $Y$ . Then  $X$  gets more points than  $Y$ , so  $Y$  does not win using Borda.

**Question 7.** Please use this example to explain why ranked choice voting does not satisfy the Condorcet Winner Criterion.

3	4	2
A	B	C
C	C	A
B	A	B

Using RCV:

Round 1: A: 3  
B: 4  
~~C: 2~~

Round 2: A: 5  
B: 4      A wins.

Condorcet:

A  $\succ$  B  
A: 3+2  
B: 4

A  $\succ$  C  
A: 3  
C: 4+2

B  $\succ$  C  
B: 4  
C: 3+2

C wins

The RCV winner is different from the Condorcet winner, so RCV does not satisfy CWC.

**Question 8.** Please prove that the Borda count satisfies the monotonicity criterion.

Imagine  $X$  is the winner using Borda, and then we boost  $X$  on some ballots. Then  $X$  will get even more points, so  $X$  still wins.

Question 9. Please explain why dictatorship satisfies IIA.

Imagine  $X$  is the winner using dictatorship, and then we change ballots without moving anyone past  $X$ .

Then  $X$  is still ranked 1<sup>st</sup> by the dictator,

so  $X$  still wins.

Question 10. Use this sample election to show how some of the voters can manipulate the election if we're using plurality. Write some words explaining why your example qualifies as a manipulation.

4	3	2
A	C	B
C	A	C
B	B	A

OG result: A: 4  
 B: 2      A wins!  
 C: 3

The  $\begin{matrix} B \\ C \\ A \end{matrix}$  voters can get a better result if they vote  $\begin{matrix} C \\ B \\ A \end{matrix}$ , in which case  $C$  will win. Since this is a better outcome for these voters, this is a manipulation.

Question 11. Using the election from Question 1, please give the probability for each candidate to win if we use the random dictator method. (Write your probabilities as fractions— you don't need to convert to percentages.)

4	4	1	2	
A	B	B	C	11 total votes
B	C	A	A	
C	A	C	B	

A: 4/11
B: 5/11
C: 2/11

**Question 12.** Use the election from Question 1, and assume we are using approval voting where each candidate approves of their top two choices. Please find the results of the election.

	4	4	1	2	
A	✓		✓	✓	A: $4+1+2 = 7$
B	✓	✓	✓		B: $4+4+1 = 9$
C		✓		✓	C: $4+2 = 6$

B wins!

**Question 13.** For each part, use this weighted voting system:  $[25 : 17, 12, 3, 3, 1, 1]$

- a) Identify any dictators, or say that there are none.
- b) Identify any voters with veto power, or say that there are none.
- c) Identify any dummies, or say that there are none.

a) No dictator

b) 17 has veto power, none of the others do

c) No dummies. Even 1 can be relevant,

like  $17 + 3 + 3 + 1 + 1 = 25$

**Question 14.** Please find the Shapley-Shubik power index for  $[25 : 12, 12, 10]$ .

perms	weights	pivot=1
ABC	12 12 <u>10</u>	C
ACB	12 10 <u>12</u>	B
BAC	12 12 <u>10</u>	C
BCA	12 10 <u>12</u>	A
CAB	10 12 <u>12</u>	B
CBA	10 12 <u>12</u>	A

A:  $2/6$

B:  $2/6$

C:  $2/6$



**Question 15.** Please find the Banzhaf power index for [25 : 12, 12, 10].

Combos			total	crit.		
A	B	C		A	B	C
A	B	C	34	✓	✓	✓
A	B		24			
A		C	24			
	B	C	22			
A			12			
	B		12			
		C	10			
∅			0			

$$A: \frac{1}{3}$$

$$B: \frac{1}{3}$$

$$C: \frac{1}{3}$$

**Question 16.** We have a population of 100 voters: 40 of them are Democrat, and 60 are Republican. We are going to divide them into 5 districts of 20 voters each. Assuming no districts are tied, please determine all the possible outcomes for how many districts can be won by each party.

20 per district, so the threshold will be 11.

D's have 40, so they can win up to 3

R's have 60, so they can win up to 5

0D/5R

1D/4R

2D/3R

3D/2R

~~4D/1R~~

~~5D/0R~~

so the possible outcomes are:

0D/5R

1D/4R

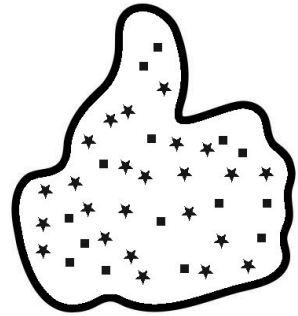
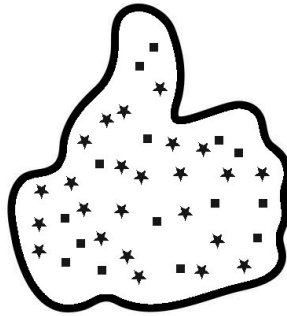
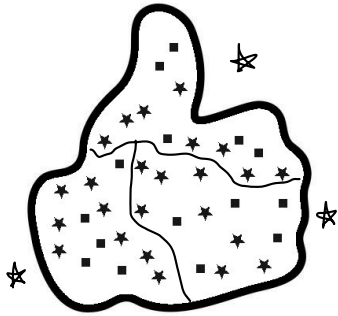
2D/3R

3D/2R

**Question 17.** This picture has 15 squares and 24 circles. I want to divide them into 3 districts of 13 voters each. (In each part, I am giving you the picture 3 times in case you mess up. If you do it right, you should only need 1 for each part.)

a) Please draw districts so that the circles have a majority in 3, and the squares have a majority in 0.

*threshold is 8*



b) Please draw districts so that the circles have a majority in ~~3~~<sup>2</sup>, and the squares have a majority in ~~0~~<sup>2</sup>.

