

Name: _____

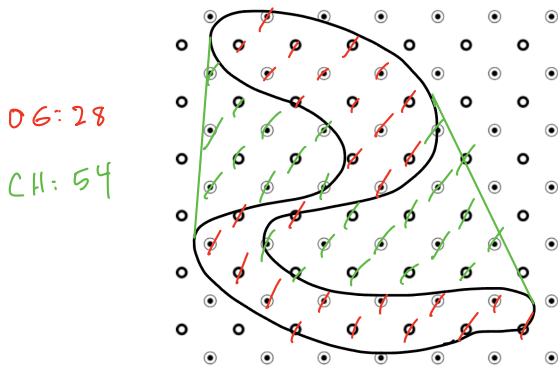
Math 1015: Exam #3

Question 18. Please find the efficiency gap for these votes between the D's and the R's. Your answer should say, "The EG is ??? in favor of ???" You can use an unsimplified fraction in your answer.

district	R votes	D votes	<u>total</u>	<u>thres</u>	<u>R wasted</u>	<u>D wasted</u>
1	22	9	31	16	6	9
2	20	12	32	17	3	12
3	6	25	31	16	6	9
4	14	16	30	16	14	0
			<u>124</u>		<u>29</u>	<u>30</u>

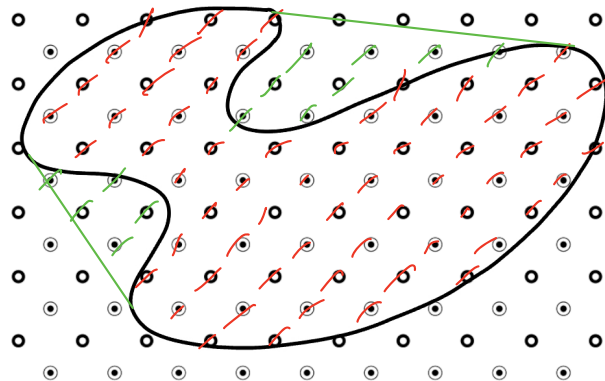
EG is $\frac{30-29}{124} = \frac{1}{124}$ in favor of R.

Question 19. Please find the convex hull ratio of each shape. You can leave your answer as an unsimplified fraction.



OG: 28
CH: 54

CHR: $\frac{28}{54}$



OG: 61
CH: 74

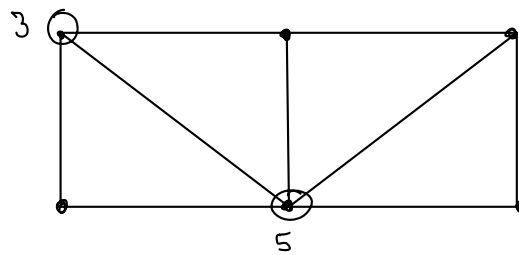
CHR: $\frac{61}{74}$

Question 20. Please find the isoperimetric quotient of each shape. You can leave your answer as an unsimplified fraction.

$A = 13$
 $P = 22$
 $IQ = \frac{4\pi \cdot 13}{22^2}$

$A = 19$
 $P = 20$
 $IQ = \frac{4\pi \cdot 19}{20^2}$

Question 21. a) Please draw an example of a graph which has at least one vertex of degree 5, and another vertex of degree 3 (there will be other vertices too). Circle the one with degree 5 and the one with degree 3.



b) Is your graph connected? Say why in a few words.

Yes mine is connected - any 2 verbs can be joined by a path.

Question 22. Imagine a graph where each vertex is a person, and two people are connected if they have ever personally communicated directly with each other by text, email, DM, etc.

- a) Who do you think would have a higher degree in this graph: you, or a toddler? Explain why in a few words.

Me! A toddler doesn't text with as many people as I do.

- b) Of these choices, which do you think would be a reasonable number for your degree in this graph: 2, 2,000, 2,000,000? Say a few words about which seems most reasonable, or explain if you think none of them are close.

This would be the total # of people I have ever texted or emailed, etc.

Of these options 2,000 seems most reasonable.

- c) Please explain in a few words what it means for two people to be connected by an edge in this graph, vs two people being connected by a path in this graph. Is there a difference?

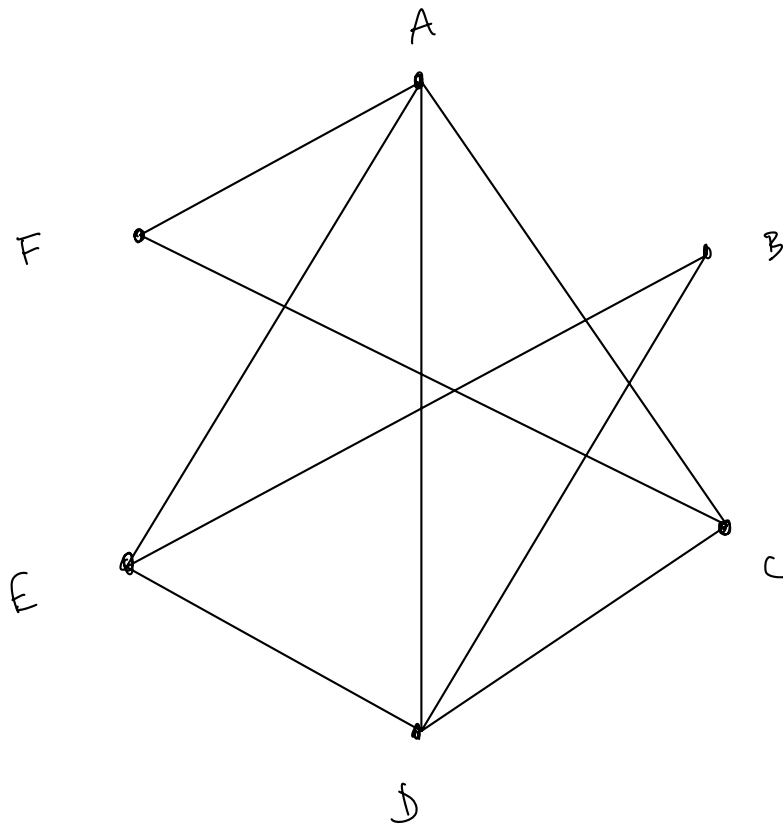
Edge means they texted directly.

A path would mean that there is a chain of texts from one person to another, then to another, etc, eventually ending up at the other person.

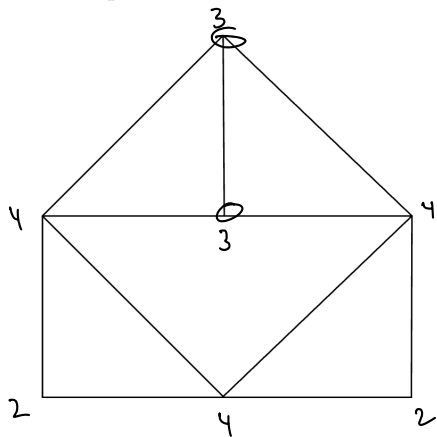
So they are quite different.

Question 23. Please draw the graph which has this formal description:

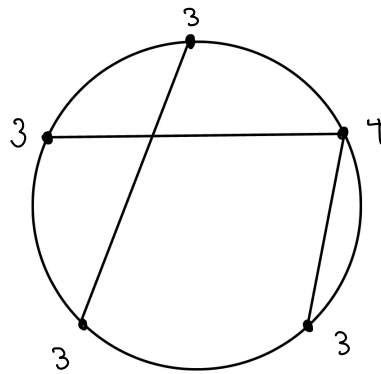
vertices: $\{A, B, C, D, E, F\}$ edges: $\{(A, C), (A, D), (A, E), (A, F), (B, D), (B, E), (C, D), (C, F), (D, E)\}$



Question 24. For each of these graphs, please say whether or not it has an Euler circuit, and whether or not it has an Euler path.



this has no E. circuit,
It has an E. path



this has neither

Old ones answers

Question 1. Please find the results of this election using plurality:

4	4	1	2
A	B	B	C
B	C	A	A
C	A	C	B

A: 4

B: 5

C: 2

B wins.

Question 2. Use the election from Question 1 and find the results using ranked choice voting.

Round 1:

A: 4

B: 5

~~C: 2~~

Rd 2:

4	4	1	2
A	B	B	A
B	A	A	B

A: 6

B: 5

A wins

Question 3. Use the election from Question 1 and find the results using Condorcet's method.

A vs B

A: 4 + 2

B: 4 + 1

A vs C

A: 3 + 1

C: 4 + 2

B vs C

B: 4 + 4 + 1

C: 2

No Condorcet winner.

Question 4. Use the election from Question 1 and find the results using Borda count.

	4	4	1	2
2	A	B	B	C
1	B	C	A	A
0	C	A	C	B

A: $4 \cdot 2 + 4 \cdot 0 + 1 \cdot 1 + 2 \cdot 1$

$8 + 1 + 2 = 11$

B: $4 \cdot 1 + 4 \cdot 2 + 1 \cdot 2$

$4 + 8 + 2 = 14$

C: $4 \cdot 0 + 4 \cdot 1 + 1 \cdot 0 + 2 \cdot 2$

$4 + 4 = 8$

B wins

Question 5. Please prove that the plurality method satisfies the majority criterion.

Imagine that a majority of voters rank X first.
Then X has more 1st place rankings than anyone else, so X wins.

Question 6. Please prove that ranked choice voting satisfies the unanimity criterion.

Imagine all voters rank X above Y. Then Y will be eliminated before X, so Y does not win.

Question 7. Please use this example to show that the plurality system does not satisfy the Condorcet winner criterion. (Write at least 1 sentence of explanation— don't just write a bunch of numbers.)

3	4	2
A	B	C
C	C	A
B	A	B

plurality: A: 3
 B: 4 B wins with plurality
 C: 2

Condorcet:

A vs B	A vs C	B vs C	C wins
A: 3+2	A: 3	B: 4	
B: 4	C: 4+2	C: 3+2	

The plurality winner is different from the Condorcet winner,
so plurality does not satisfy CWC.

Question 8. Please prove that the plurality system satisfies monotonicity.

Imagine X wins using plurality, and then we boost X on some ballots. Then X can only get more votes in 1st position, so X will still win.

Question 9. Please prove that Condorcet's method satisfies IIA.

Imagine X is the Condorcet winner, and then we change ballots without moving anyone past X . Then X still wins all their pair matchups, so X still wins using Condorcet's method.

Question 10. Use this sample election to show how some of the voters can manipulate the election if we're using Borda. Write some words explaining why your example qualifies as a manipulation.

	6	3	2
2	A	C	B
1	C	A	C
0	B	B	A

If $\begin{matrix} B \\ C \\ A \end{matrix}$ changes to $\begin{matrix} C \\ A \\ B \end{matrix}$,

OG result:

$$A: 6 \cdot 2 + 3 \cdot 1 + 2 \cdot 0 = 15$$

$$B: 6 \cdot 0 + 3 \cdot 0 + 2 \cdot 2 = 4$$

$$C: 6 \cdot 1 + 3 \cdot 2 + 2 \cdot 1 = 14$$

A wins!

this gives C 2 more points,
B 2 less points,

so if all be

A: 15
B: 2
C: 16

Now C wins!
This is a preferable result for $\begin{matrix} B \\ C \\ A \end{matrix}$ voters,
so this is a manipulation.

Question 11. I told my friend that I like the random dictator method, but he said it sounds stupid because if you're just going to choose the winner at random, then there's no point in even voting at all. Do you agree with this objection? How would you respond?

This isn't a good objection - in Random Dictator method, the result isn't entirely random. It still depends statistically on who got the most votes. So in this method the votes really do matter.

Question 12. Use the election from Question 1, and assume we are using approval voting where each candidate approves of their top two choices. Please find the results of the election.

	4	4	1	2	
A	✓		✓	✓	A: $4+1+2=7$
B	✓	✓	✓		B: $4+4+1=9$
C		✓		✓	C: $4+2=6$

B wins!

Question 13. For each part, use this weighted voting system: $[30 : 10, 10, 10, 5, 5, 4]$

- a) Identify any dictators, or say that there are none.
- b) Identify any voters with veto power, or say that there are none.
- c) Identify any dummies, or say that there are none.

a) none

b) none

c) 4 is a dummy.

A B C

Question 14. Please find the Shapley-Shubik power index for $[15 : 14, 8, 7]$.

<u>perms</u>	<u>weights</u>	<u>pivotal</u>	
A B C	14 <u>8</u> 7	B	A: 2/6
A C B	14 <u>7</u> 8	C	
B A C	8 <u>14</u> 7	A	B: 2/6
B C A	8 <u>7</u> 14	C	
C A B	7 <u>14</u> 8	A	C: 2/6
C B A	7 <u>8</u> 14	B	

Question 15. Please find the Banzhaf power index for $[15 : 14, 8, 7]$.

<u>Combos</u>	<u>total</u>	<u>crits</u>			
		A	B	C	
A B C	29				A: 2/6
A B	22	✓	✓		B: 2/6
A C	21	✓		✓	
B C	15		✓	✓	C: 2/6
A	14				
B	8				
C	7				
∅	0				

Question 16. We have a population of 100 voters: 40 of them are Democrat, and 60 are Republican. We are going to divide them into 5 districts of 20 voters each. Assuming no districts are tied, please determine all the possible outcomes for how many districts can be won by each party.

20 per district, so the threshold will be 11.

0D/5R

1D/4R

2D/3R

3D/2R

~~4D/1R~~

~~5D/0R~~

D's have 40, so they can win up to 3

R's have 60, so they can win up to 5

So the possible outcomes are:

0D/5R

1D/4R

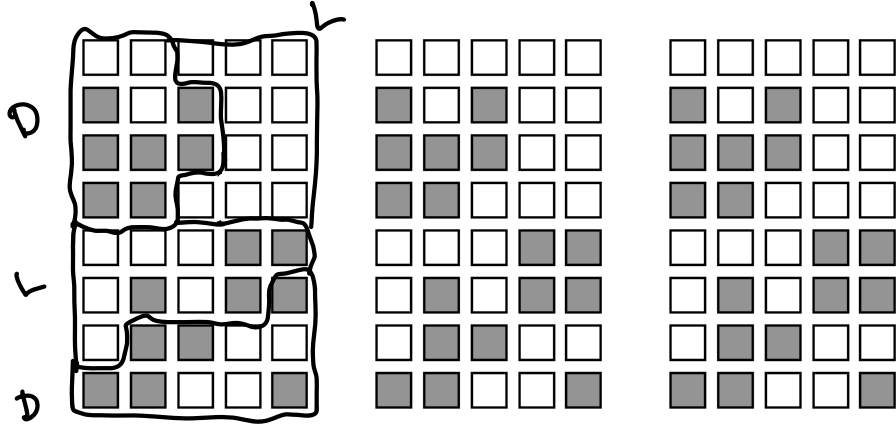
2D/3R

3D/2R.

Question 17. This grid represents a population of 40 voters. I want to divide them into 4 districts of 10 voters each. (In each part, I am giving you the picture 3 times in case you mess up. If you do it right, you should only need 1 for each part.)

- a) Please draw districts so that the dark squares have a majority in 2, and the light squares have a majority in 2.

*threshold
is 6*



- b) Please draw districts so that the dark squares have a majority in 1, and the light squares have a majority in 3.

