## Math 1015: Homework #3

Question 1. Explain why the Borda count satisfies the Unanimity criterion.

Question 2. Explain why the Condorcet method satisfies the Unanimity criterion.

Question 3. Explain why Ranked Choice Voting satisfies the Majority criterion.

**Question 4.** Give an example demonstrating how the plurality system satisfies the monotonicity criterion. (Make up an example with As, Bs, Cs, etc, and show how boosting the winner makes the outcome stay the same.)

Question 5. Explain why the Condorcet method satisfies the Monotonicity criterion.

**Question 6.** Please explain in your own words why the monotonicity property is desirable in a real-world voting system. (You only need to write a few sentences at most.)

**Question 7.** On the last page of this PDF you'll find a description of a voting method called BTR-IRV. Please find the winner using BTR-IRV for this election:

4	3	5	1
А	В	D	С
В	Α	$\mathbf{C}$	D
С	D	Α	В
D	$\mathbf{C}$	В	Α

Question 8. Explain why BTR-IRV satisfies the Condorcet Winner Criterion.

Question 9. Explain why BTR-IRV satisfies the Majority Criterion.

Here is another voting method called "Bottom-Two Runoff Instant Runoff Voting" (BTR-IRV). This is a fancier combination of RCV and the Condorcet method. We do elimination rounds like in RCV, but the elimination rule is more complicated. In each round we count the total 1st place rankings, and then we do a separate pairwise comparison between the bottom two, and eliminate whoever loses in that comparison.

Here is an example:

 $5 \ 3 \ 4$ 

- A D B
- B A C
- C C D
- D B A

**Round 1:** We count up the 1st-place totals in round 1, and we get A: 8, B: 4, C: 0, D: 3. The bottom 2 are C and D, so we now do the C vs D comparison using the full chart above. I see C: 9 and D: 3, so D loses this comparison and D is eliminated.

Round 2: After eliminating D, the votes look like this:

 $\begin{array}{cccc} 5 & 3 & 4 \\ \hline A & A & B \\ B & C & C \\ C & B & A \end{array}$ 

The 1st-place totals are now A: 8, B: 4, C: 0. The bottom 2 are B and C, so we now do the B vs C comparison. I get B; 9 and C: 3, so C is eliminated.

Round 3: After eliminating C, the votes now look like:

 $\begin{array}{ccc} 5 & 3 & 4 \\ \hline A & A & B \\ \hline \end{array}$ 

B B A

Now it's just A vs B, and we get A: 8 and B: 4, so A wins.