

Math 1015

Homework #3

#1, 3, 5, 8

#1 Borda satisfies unanimity

Proof Assume all voters rank X above Y .
Then X gets more Borda points than Y ,

thus Y does not win using Borda.

Q.E.D.

#3 RCV satisfies majority

Proof Assume X is ranked 1st by a majority
of voters.

Then, in every round, X gets more 1st-place
rankings than any other candidate, so X is
never eliminated.

Thus X is the winner using RCV.

#5 Condorcet's Method satisfies Monotonicity

Proof Assume X is the winner using Condorcet's method, and then we change ballots to boost X .

Since we boosted X , the result is that X does better in each 1 vs 1 matchup, so X still wins them all.

Thus X is still the winner using Condorcet's method.

#7

Round 1:

4	3	5	1
A	B	D	C
B	A	C	D
C	D	A	B
D	C	B	A

A: 4

B: 3

C: 1

D: 5

Bottom two are B & C

B vs C

$$B: 4 + 3 = 7$$

$$C: 5 + 1 = 6$$

C is eliminated

Round 2

4	3	5	1
A	B	D	D
B	A	A	B
D	D	B	A

A: 4

B: 3

D: 6

Bottom 2 are A & B

A vs B: $A: 4 + 5 = 9$

$$B: 3 + 1 = 4$$

B is eliminated

Round 3

$\frac{4}{A}$	$\frac{3}{A}$	$\frac{5}{D}$	$\frac{1}{D}$
D	D	A	A

A: 7
D: 6

A wins!