

# Math 1015 Homework #4

1. Imagine  $X$  is the winner using dictatorship, then we change ballots without moving anyone past  $X$ . Then  $X$  is still the top choice of the dictator, so  $X$  still wins.

2.

	4	3	3	2
4	A	B	C	D
3	B	A	D	E
2	C	C	A	C
1	D	D	B	A
0	E	E	E	B

Borda:

$$A: 4 \times 4 + 3 \times 3 + 3 \times 2 + 2 \times 1 = 33$$

$$B: 4 \times 3 + 3 \times 4 + 3 \times 1 + 2 \times 0 = 27$$

$$C: 4 \times 2 + 3 \times 2 + 3 \times 4 + 2 \times 2 = 30$$

$$D: 4 \times 1 + 3 \times 1 + 3 \times 3 + 2 \times 4 = 28$$

$$E: 4 \times 0 + 3 \times 0 + 3 \times 0 + 2 \times 3 = 6$$

A is OG winner! Now we can move votes around without moving anyone past A:

	4	3	3	2
4	A	B	C	C
3	B	A	D	E
2	C	C	A	D
1	D	D	B	A
0	E	E	E	B

A (didn't move) : 33

B " : 27

$$C: 4 \times 2 + 3 \times 2 + 3 \times 4 + 2 \times 4 = 34$$

$$D: 4 \times 1 + 3 \times 1 + 3 \times 3 + 2 \times 2 = 24$$

E: 6

Now C wins, which violates IIA.

3

	2	4	3
A		<u>B</u>	D
C		A	A
<u>B</u>		D	C
D		C	<u>B</u>

Plurality:

A: 2

B: 4

C: 0

D: 3

B is OG winner

a

D  
A  
C  
B

can change to

A  
D  
C  
B

which makes A win,

which is preferable to them.

b

A  
C  
B  
D

cannot manipulate. The only improved results they

could try for are A & C.

They already put A on top and A still lost,

so they can't make A win.

They could try to make C win but even

doing  
C  
A  
B  
D

is not enough to make C the winner.

So these ones cannot manipulate.

4. Monotonicity is about boosting the winner.

IIA is about moving the losers without anyone moving past the winner.

Manipulation is about doing any moves in one column to achieve a more preferable result.