

Math 1015: Homework #6

Question 1. Compute the Banzhaf power index for [15: 10,7,3].

Question 2. Compute the Banzhaf power index for [12: 8, 8, 4].

combos	total weight	crit		
		A	B	C
A B C	20			
A B	16	✓	✓	
A C	12	✓		✓
B C	12		✓	✓

A: 2/6
 B: 2/6
 C: 2/6

A B C D

Question 3. Compute the Banzhaf power index for [20: 10, 7, 5, 5].

combo	total	A	B	C	D
A B C D	27	✓			
A B C	22	✓	✓	✓	
A B D	22	✓	✓		✓
A C D	20	✓		✓	✓
<hr/>					
B C D	17				
A B	17				
A C	15				
A D	15				
B C	12				
B D	12				
C D	10				
A	10				
B	7				
C	5				
D	5				
∅	0				

A: $\frac{4}{10}$

B: $\frac{2}{10}$

C: $\frac{2}{10}$

D: $\frac{2}{10}$

The last page includes some extra copies of the pictures so you can practice. This whole document is at the class website in case you need to print it again.

Question 4. In this grid at the bottom of the page there are 20 light squares and 20 dark squares. We must divide the grid into 4 districts of 10 voters each.

- a) Please determine which outcomes are possible for how many districts can be won by the lights and the darks, and which are impossible. (You should be able to do this without looking at the pictures- all you need to know are the numbers above.) Say which ones are possible, and explain why the impossible ones are impossible.

10 voters per district, so we need 6 voters to win each dist.

L has 20 voters, so L can win up to 3 districts ($3 \times 6 = 18$)

Similarly, D can win up to 3.

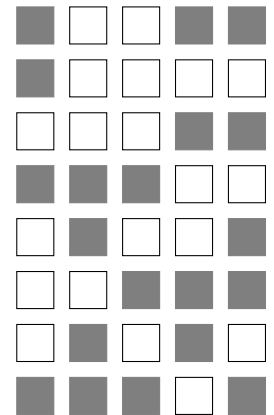
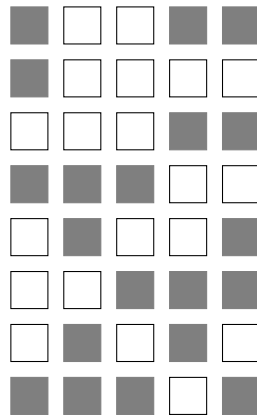
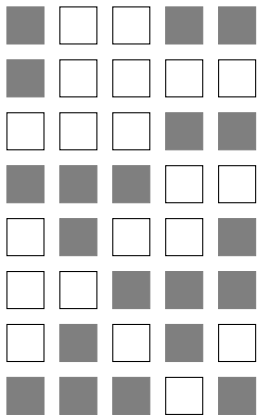
So the possibilities are:

3L	1D
2L	2D
1L	3D

~~4L 0D~~
~~3L 1D~~
~~2L 2D~~
~~1L 3D~~
~~0L 4D~~

- b) For each of the possible outcomes that you identified in the first part, show how you can make districts which realize that outcome.

Clearly label which picture gives which outcome.

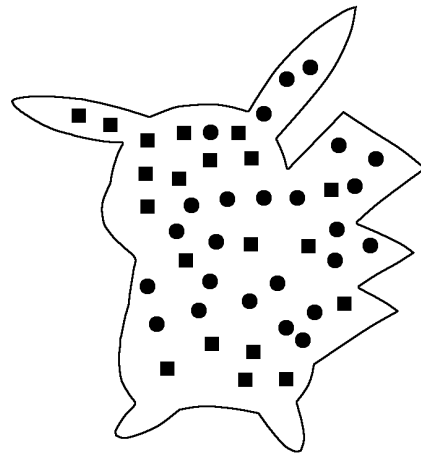
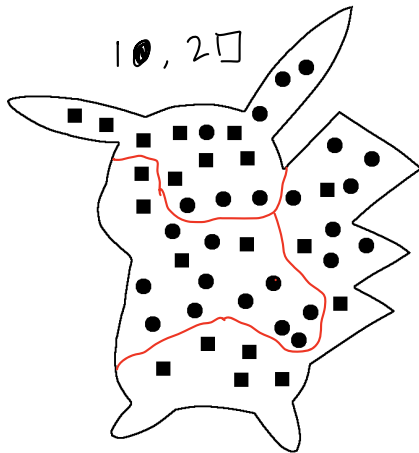
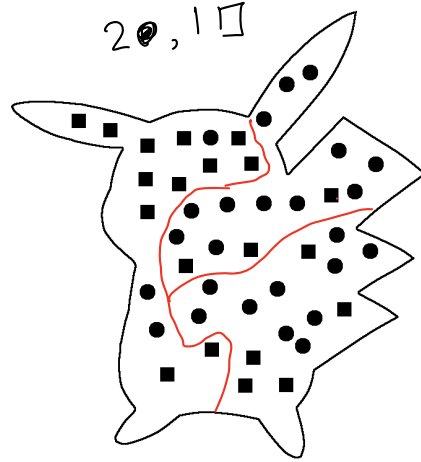
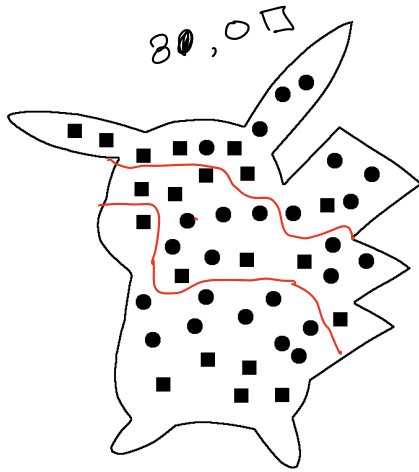


Question 5. Pikachuland has 45 inhabitants: 20 squares and 25 circles. We need to divide them into 3 districts of 15 voters each.

a) Please determine all possible outcomes that can be achieved by districting, and draw diagrams for all the possible ones. (You may not need to use all of these pictures.) Clearly label which picture gives which result.

thresh = 8

□ can win up to 2
 ● can win up to 3



Many possible answers!

b) For all outcomes that you did not draw, explain why they are impossible.

