Name: _____

Math 1015: Homework #8

Question 1. a) Find the isoperimetric quotient of each of these 4 districts. (I gave you the picture 4 times so you can draw on one for each district.)

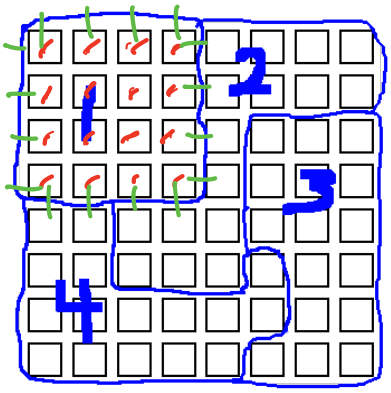
#1

A = 16

P = 16

$IQ = \frac{4\pi \cdot 16}{16^2}$

$= .78$



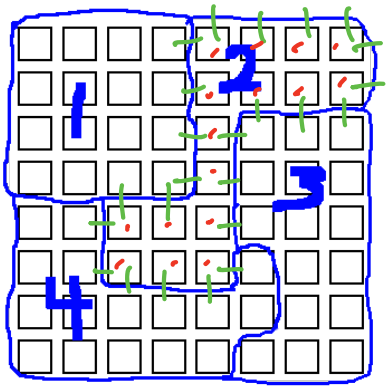
#2

A = 16

P = 24

$IQ = \frac{4\pi \cdot 16}{24^2}$

$= .34$

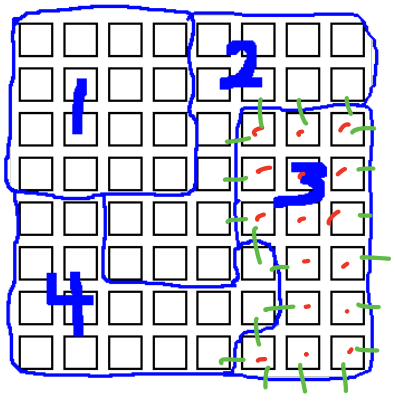


#3

A = 16

P = 20

IQ = .50

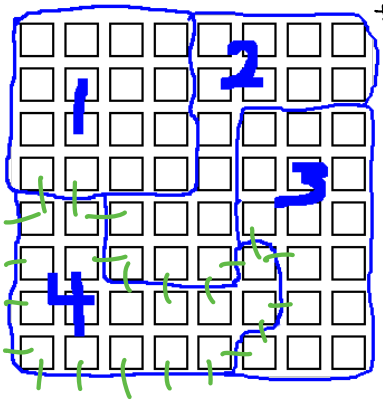


#4

A = 16

P = 22

IQ = .41



b) According to the isoperimetric quotient, which shape is the weirdest (use a calculator so you can compare the values)? Which is the least weird?

Weirdest is # 2

least weird is # 1

Question 2. In each part, draw a single shape having the properties described.

a) High convex hull ratio and high isoperimetric quotient.

b) High convex hull ratio and low isoperimetric quotient.

c) Low convex hull ratio and low isoperimetric quotient.

Question 3. a) Draw an example of a graph having 8 vertices and 13 edges.

b) Next to each vertex, write its degree.

c) Say if your graph is connected or not.

Question 4. My daughter played little league softball in the town of Fairfield. There were 8 teams in her age-group, with 12 girls on each team (each girl is on only 1 team). Imagine a graph where each vertex is a girl, and two vertices are connected by an edge if those two girls are on the same team.

a) What is the degree of my daughter's vertex in this graph?

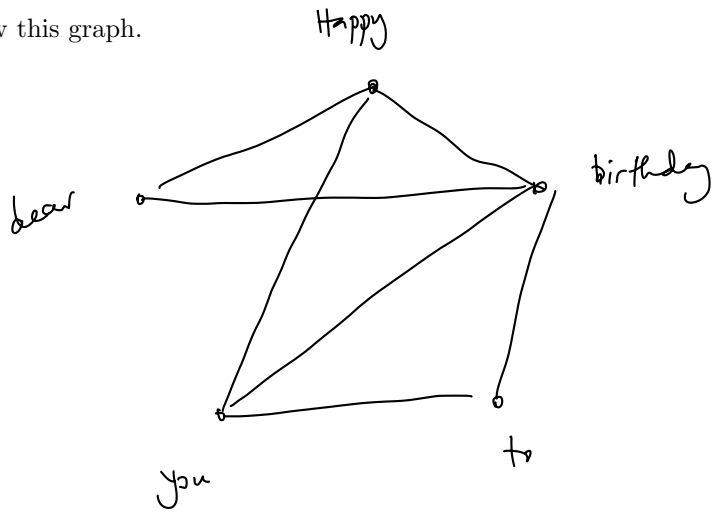
11 - there are 11 other girls on her team, so she has 11 edges in the graph.

b) Is the graph connected? Say why or why not.

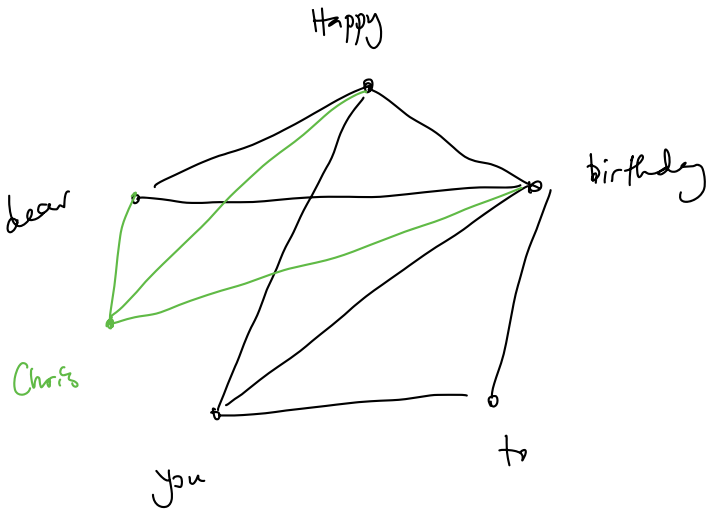
No - there are 8 teams, and no connections between the teams.

Question 5. Imagine a graph where each vertex is a word in the "Happy birthday to you" song, ignoring the person's name. (There should be 5 vertices.) Two vertices are connected by an edge when the two words have some letter in common.

a) Draw this graph.



b) Redraw the graph, but with your own name added into the song.



c) Instead of your name, think of a name that we could add that makes the graph become disconnected. (You don't have to use a real name— you can just make up some jumble of letters that would work.)

qfwx

Question 6. Imagine a graph where each vertex is one of the 48 states of the continental USA, and two states are connected by an edge when they share a land border. (Look at a map to answer some of these questions.)

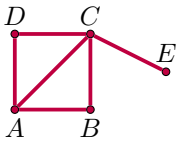
a) Which states have the smallest and largest degrees? (Hint: the largest degree is 9)

b) Is this graph connected?

c) Do your answers to a) and b) change if we also allow states to be connected by water? (Either the Atlantic or Pacific Ocean.) Explain what would be different.

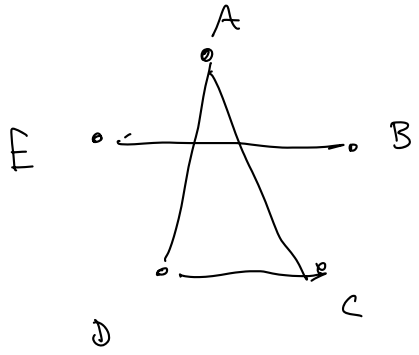
d) Do your answers to a) and b) change if we also include Alaska & Hawaii? (But do not include changes from part c)

Question 7. Please write the formal definition of this graph as sets of vertices and edges:



Question 8. a) Please draw a picture of the graph formally defined by:

$$\text{vertices} : \{A, B, C, D, E\}, \qquad \text{edges} : \{(A, D), (A, C), (C, D), (B, E)\}$$



b) Is this graph connected? Explain specifically according to the definition of connectedness. (You should say something about paths.)

Its not connected, since there's no path from E to A.