Math 1172 HW #6

7.3 # 7/13, 10/16

7.4. # 9, 18

#7/13 $\displaystyle\int_0^a \frac{1}{(a^2+x^2)^{3/2}}\,dx$

$x = a\tan\theta$

$dx = a\sec^2\theta\,d\theta$

change values:

$x=a$: $a = a\tan\theta$, $\tan\theta = 1$, $\theta = \pi/4$

$x=0$: $0 = a\tan\theta$, $\tan\theta = 0$, $\theta = 0$

$$\int_0^{\pi/4} \frac{1}{(a^2+a^2\tan^2\theta)^{3/2}} \cdot a\sec^2\theta\,d\theta$$

$$= \int_0^{\pi/4} \frac{1}{a^3(1+\tan^2\theta)^{3/2}}\, a\sec^2\theta\,d\theta$$

$$= \frac{1}{a^2}\int_0^{\pi/4} \frac{\sec^2\theta}{(\sec^2\theta)^{3/2}}\,d\theta = \frac{1}{a^2}\int_0^{\pi/4} \frac{\sec^2\theta}{\sec^3\theta}\,d\theta$$

$$= \frac{1}{a^2}\int_0^{\pi/4} \cos\theta\,d\theta = \frac{1}{a^2}\cdot \sin\theta\Big|_0^{\pi/4} = \frac{1}{a^2}\cdot\frac{1}{\sqrt{2}}$$

#10/16

$$\int_0^{2/3} \sqrt{4-9x^2}\,dx = \int_0^{2/3} 3\sqrt{4/9 - x^2}\,dx$$

$x = \frac{2}{3}\sin\theta$

$dx = \frac{2}{3}\cos\theta\,d\theta$

change boundaries:

$x = 2/3$: $\frac{2}{3} = \frac{2}{3}\sin\theta$ $\quad \sin\theta = 1$ $\quad \theta = \pi/2$

$x = 0$: $0 = \frac{2}{3}\sin\theta$ $\quad \sin\theta = 0$ $\quad \theta = 0$

$$3\int_0^{\pi/2} \sqrt{4/9 - \left(\tfrac{2}{3}\sin\theta\right)^2}\cdot\frac{2}{3}\cos\theta\,d\theta = 3\int_0^{\pi/2} \frac{2}{3}\sqrt{1-\sin^2\theta}\cdot\frac{2}{3}\cos\theta\,d\theta$$

$$= \frac{4}{3}\int_0^{\pi/2} \sqrt{\cos^2\theta}\cdot\cos\theta\,d\theta = \frac{4}{3}\int_0^{\pi/2} \cos^2\theta\,d\theta$$

$$= \frac{4}{3}\int_0^{\pi/2} \frac{1}{2}(1+\cos 2\theta)\,d\theta = \frac{2}{3}\left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/2}$$

$$= \frac{2}{3}\left(\pi/2 + \frac{1}{2}\sin\pi\right) - \frac{2}{3}\left(0 + \frac{1}{2}\sin 0\right)$$

$$= \frac{2}{3}\cdot\pi/2 = \pi/3$$

7.4 #9

$$\int \frac{5x+1}{(2x+1)(x-1)}\,dx \qquad \frac{5x+1}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1}$$

$$= Ax - A + 2Bx + B$$

$$= (A+2B)x - A+B$$

$5 = A+2B$

$1 = -A+B$

$6 = 3B$

$B = 2$

$1 = -A+2$

$A = 1$

$$\int \frac{1}{2x+1} + \frac{2}{x-1}\,dx$$

$$= \tfrac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$$

#18

$$\int_1^2 \frac{3x^2+6x+2}{x^2+3x+2}\,dx$$

$$x^2+3x+2 \;\overline{)\;3x^2+6x+2}$$

$$3x^2+9x+6$$

$$-3x-4$$

$$= \int_1^2 3 + \frac{-3x-4}{x^2+3x+2}\,dx \qquad \frac{-3x-4}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$-3x-4 = Ax+2A+Bx+B$$

$$= (A+B)x + 2A+B$$

$-3 = A + B$ $\qquad -3 = -1 + B$

$-4 = 2A + B$ $\qquad B = -2$

$1 = -A$

$A = -1$

$$= \int_1^2 3 + \frac{-1}{x+1} + \frac{-2}{x+2}\, dx$$

$$= 3x - \ln|x+1| - 2\ln|x+2| \Big|_1^2$$

$$= 6 - \ln 3 - 2\ln 4 - \left(3 - \ln 2 - 2\ln 3\right)$$