Math 1172

Homework #7

Section 7.8 #14/22, #15/23, #20/28, #31/39

14/22

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{e^{-1/x}}{x^2}\,dx \qquad \begin{aligned} u &= -1/x = -x^{-1} \\ du &= x^{-2}\,dx \end{aligned}$$

$$= \lim_{t\to\infty}\int e^u\,du = \lim_{t\to\infty} e^u \Big|_{x=1}^{x=t}$$

$$= \lim_{t\to\infty} e^{-1/x}\Big|_1^t = \lim_{t\to\infty} e^{-1/t} - e^{-1}$$

$$= e^0 - e^{-1} = 1 - \frac{1}{e}$$

15/23

$$\lim_{t\to\infty}\int_0^t \sin^2\alpha\,d\alpha = \lim_{t\to\infty}\int_0^t \tfrac{1}{2}(1-\cos 2\alpha)\,d\alpha$$

$$= \frac{1}{2}\lim_{t\to\infty} \alpha - \tfrac{1}{2}\sin 2\alpha \Big|_0^t$$

$$= \frac{1}{2}\left(\lim_{t\to\infty} t - \tfrac{1}{2}\sin 2t - \left(0 - \tfrac{1}{2}\sin 2\cdot 0\right)\right)$$

$$= \frac{1}{2}\left(\lim_{t\to\infty} \underset{\downarrow\atop\infty}{t} - \tfrac{1}{2}\sin 2t\right) \qquad \text{diverges!}$$

#20/28

$$\lim_{t\to\infty} \int_2^t y e^{-3y}\, dy \qquad u = y \quad du = dy \qquad dv = e^{-3y} dy \quad v = -\frac{1}{3} e^{-3y}$$

$$= \lim_{t\to\infty} -\frac{1}{3} y e^{-3y} - \int_2^t -\frac{1}{3} e^{-3y}\, dy$$

$$= \lim_{t\to\infty} -\frac{1}{3} y e^{-3y} + \frac{1}{3} \int_2^t e^{-3y}\, dy$$

$$= \lim_{t\to\infty} -\frac{1}{3} y e^{-3y} - \frac{1}{9} e^{-3y} \Big|_2^t$$

$$= \lim_{t\to\infty} \underbrace{-\frac{1}{3} t e^{-3t}}_{\to 0} - \underbrace{\frac{1}{9} e^{-3t}}_{\to 0} - \left(\frac{-1}{3} \cdot 2 \cdot e^{-6} - \frac{1}{9} e^{-6} \right)$$

by this

$$\lim_{t\to\infty} \frac{-1}{3} t e^{-3t} = \lim_{t\to\infty} -\frac{1}{3} \frac{t}{e^{3t}} \to \frac{\infty}{\infty}$$

$$= \frac{-1}{3} \lim_{t\to\infty} \frac{1}{3e^{3t}} = 0$$

So the answer is:

$$-\left(\frac{-1}{3} \cdot 2 \cdot e^{-6} - \frac{1}{9} e^{-6} \right)$$

#31|39

$$\int_{-2}^{3} \frac{1}{x^4}\,dx \qquad \text{vert 'tote at} \quad x=0$$

$$= \lim_{t\to 0} \int_{-2}^{t} x^{-4}\,dx + \lim_{t\to 0} \int_{t}^{3} x^{-4}\,dx$$

first one:

$$\lim_{t\to 0} \int_{-2}^{t} x^{-4}\,dx = \lim_{t\to 0} \left. -\frac{1}{3}x^{-3} \right|_{-2}^{t} = \lim_{t\to 0} \frac{-1}{3}t^{-3} - \frac{-1}{3}(-2)^{-3}$$

$$= \lim_{t\to 0} \frac{-1}{3}\frac{1}{t^3} + \frac{1}{3}\cdot(-2)^{-3}$$

$$\downarrow$$

$\frac{1}{0}$ Does not exist!

So the original integral diverges.