Math 1172

Homework #8

Section 8.1 #14/16, #37/51

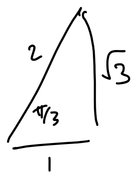
Section 11.1 #10/14, #27/33

8.1 #14/16

$y = \ln(\cos x)$ $\qquad \frac{dy}{dx} = \frac{1}{\cos x} \cdot \sin x = \tan x$

$$\int_0^{\pi/3} \sqrt{1+(\tan x)^2}\, dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} \sec x \, dx$$

$$= \ln\left|\sec x + \tan x\right| \Big|_0^{\pi/3} = \ln\left|\sec \tfrac{\pi}{3} + \tan \tfrac{\pi}{3}\right| - \ln\left|\sec 0 + \tan 0\right|$$



$$= \ln\left|2+\sqrt{3}\right| - \ln\left|1+0\right| = \ln\left|2+\sqrt{3}\right|$$

8.1 #37/51

$f(x) = \frac{1}{4}e^x + e^{-x}$

area under the curve: $\int_a^b \frac{1}{4}e^x + e^{-x}\, dx$

arclength: $\int_a^b \sqrt{1+(f'(x))^2}\, dx = \int_a^b \sqrt{1+\left(\frac{1}{4}e^x - e^{-x}\right)^2}\, dx$

$$= \int_a^b \sqrt{1+\left(\frac{1}{16}e^{2x} - \frac{1}{2} + e^{-2x}\right)}\, dx = \int_a^b \sqrt{\frac{1}{16}e^{2x} + \frac{1}{2} + e^{-2x}}\, dx$$

$$= \int_a^b \sqrt{\left(\frac{1}{4}e^x + e^{-x}\right)^2}\, dx = \int_a^b \frac{1}{4}e^x + e^{-x}\, dx$$

the same!

11.1 #10/14

$a_1 = 6 \qquad a_{n+1} = \frac{a_n}{n}$

$a_1 = 6$

$a_2 = \frac{6}{1} = 6$

$a_3 = \frac{6}{2} = 3$

$a_4 = \frac{3}{3} = 1$

$a_5 = \frac{1}{4}$

11.1 # 27/33

$$\lim_{n\to\infty} 3^n 7^{-n} = \lim_{n\to\infty} \left(\frac{3}{7}\right)^n = \lim_{x\to\infty} \left(\frac{3}{7}\right)^x = 0$$

since $3/7 < 1$.