Math 1172

Homework #10

Section 11.3 #15/17, #21/23

Section 11.4 #12/16, #37/45

11.3 # 15/17

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2}$$

$$\int_1^{\infty} \frac{\sqrt{x}+4}{x^2}\,dx = \int_1^{\infty} x^{-2}(x^{1/2}+4)\,dx = \int_1^{\infty} x^{-1.5} + 4x^{-2}\,dx$$

$$= \lim_{t\to\infty} \frac{1}{-.5}x^{-.5} + 4\cdot\frac{1}{-1}x^{-1}\Big|_1^t = \lim_{t\to\infty} -2x^{-.5} - 4x^{-1}\Big|_1^t$$

$$= \lim_{t\to\infty} -2t^{-.5} - 4t^{-1} - \left(-2\cdot 1^{-.5} - 4\cdot 1^{-1}\right)$$

$$= 0 \quad 0 \quad - (-2-4) = 6$$ converges

11.3 #21/23

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

$$\int_2^{\infty} \frac{1}{x\ln x}\,dx \qquad u = \ln x \quad du = \frac{1}{x}\,dx$$

$$= \int \frac{1}{u}\,du = \ln|u| = \ln|\ln x|\Big|_2^{\infty}$$

$$= \lim_{t\to\infty} \ln|\ln x|\Big|_2^t = \lim_{t\to\infty} \underbrace{\ln|\ln t|}_{\to\infty} - \ln|\ln 2|$$ diverges

Section 11.4 # 12/16

$$\sum \frac{6^n}{5^n - 1} \qquad \text{limit comparison with} \quad \frac{6^n}{5^n}$$

$$\lim_{n\to\infty} \frac{6^n/5^n-1}{6^n/5^n} = \lim_{n\to\infty} \frac{5^n}{5^n-1} \overset{H}{=} \lim_{n\to\infty} \frac{5^n \ln 5}{5^n \ln 5} = 1 > 0.$$

since $\sum \frac{6^n}{5^n} = \sum \left(\frac{6}{5}\right)^n$ is geometric with $r = \frac{6}{5} > 1$,

this diverges, so $\sum \frac{6^n}{5^n - 1}$ also diverges.

Section 11.4 # 37/45

$$\sum \frac{d_n}{10^n} \qquad \text{compare with} \quad \frac{9}{10^n}:$$

$\frac{d_n}{10^n} < \frac{9}{10^n}$, since d_n is a single digit.

and $\sum \frac{9}{10^n} = 9 \sum \frac{1}{10^n}$ is geometric with $r = \frac{1}{10}$,

so it converges.

So $\sum_{n=1}^{\infty} \frac{d_n}{10^n}$ converges.