Math 1172 Homework #11

Section 11.5 #6, #8

Section 11.6 #6/10, #22/26

11.5 #6 $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$ converges

we need to show $b_n = \frac{1}{\sqrt{n+1}}$ is decreasing and $\to$ to 0.

decreasing: We need $b_{n+1} < b_n$

WTS $\frac{1}{\sqrt{n+2}} < \frac{1}{\sqrt{n+1}}$, i.e. $\sqrt{n+1} < \sqrt{n+2}$

this is true since $n+1 < n+2$.

limit: $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$

So it converges.

11.5 #8 $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2+n+1}$ This diverges.

We'll show $\lim a_n \neq 0$, which means it diverges.

$$\lim_{n\to\infty} \underbrace{(-1)^n}_{\text{alternates}} \underbrace{\frac{n^2}{n^2+n+1}}_{1}$$

So this limit is not 0.

11.6 #6 (old edition)

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+4}$$

Ratio test:

$$\lim_{n\to\infty} \left| \frac{(-1)^n \cdot (n+1)}{(n+1)^2+4} \cdot (-1)^{n-1} \frac{n^2+4}{n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)(n^2+4)}{(n^2+2n+5)n} \right| = 1$$ inconclusive.

Alt. series test: $b_n = \frac{n}{n^2+4}$

decreasing: WTS $b_{n+1} < b_n$, i.e. $\frac{n+1}{(n+1)^2+4} < \frac{n}{n^2+4}$

WB $\frac{n+1}{n^2+2n+5} < \frac{n}{n^2+4}$

WTS $(n+1)(n^2+4) < n(n^2+2n+5)$

WTS $\cancel{n^3}+n^2+4n+4 < \cancel{n^3}+2n^2+5n$

WB $4 < n^2+n$, True for all n beyond $n=1$.

limit: $\lim \frac{n}{n^2+4} = 0$

So it converges!

Does it converge absolutely?

$$\sum |a_n| = \sum \frac{n}{n^2+4}$$

limit comp with $\frac{1}{n}$ shows this diverges.

So it is conditionally convergent.

11.6 # 10 (new edition)

$$\sum_{n=1}^{\infty} \frac{n!}{100^n}$$ ratio test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)!}{100^{n+1}} \cdot \frac{100^n}{n!} \right| = \lim_{n\to\infty} \left| (n+1) \cdot \frac{1}{100} \right| \to \infty$$

so it diverges.

11.6 # 22 (old version)

$$\sum \frac{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-1)}{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}$$ ratio test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{2} \cdot \cancel{5} \cdot \ldots \cdot (3(n+1)-1)}{\cancel{3} \cdot \cancel{5} \cdot \ldots (2(n+1)+1)} \cdot \frac{\cancel{3} \cdot \cancel{5} \cdot \ldots \cdot (2n+1)}{\cancel{2} \cdot \cancel{5} \cdot \ldots \cdot (3n-1)} \right|$$

$$= \lim_{n\to\infty} \left| \frac{3(n+1)-1}{2(n+1)+1} \right| = \lim_{n\to\infty} \left| \frac{3n+2}{2n+3} \right| = \frac{3}{2} > 1$$

so it diverges.

11.6 # 26 (new version)

$$\sum_{n=0}^{\infty} (\arctan n)^n$$ root test:

$$\lim_{n\to\infty} \left| \sqrt[n]{(\arctan n)^n} \right| = \lim_{n\to\infty} \left| \arctan n \right| = \pi/2 > 1$$

so it diverges.