Math 1172

Homework # 12

Section 11.8 #9/15, 24/32

Section 11.9 #4/6, 8/10

11.8 #9/15

$$\sum \frac{x^n}{n^4 \cdot 4^n}$$

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^4 \cdot 4^{n+1}} \cdot \frac{n^4 \cdot 4^n}{x^n} \right|$$

$$= \lim_{n\to\infty} |x| \cdot \frac{1}{4} \cdot \left(\frac{n}{n+1}\right)^4 = \frac{1}{4}|x|$$

$$\frac{1}{4}|x| < 1$$

$|x| < 4$ rad. of conv $= 4$

converges on $(-4, 4)$

Endpts:

$x = 4$: $\sum \frac{4^n}{n^4 \cdot 4^n} = \sum \frac{1}{n^4}$ p-series, converges

$x = -4$: $\sum \frac{(-4)^n}{n^4 \cdot 4^n} = \sum \frac{(-1)^n}{n^4}$ Converges by alt. series test.

interval of convergence: $[-4, 4]$

11.8 # 24/32

$$\sum \frac{n^2 x^n}{2\cdot 4\cdot 6\cdot \ldots \cdot (2n)}$$

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2 x^{n+1}}{2\cdot 4 \cdot \ldots \cdot 2n \cdot (2n+2)} \cdot \frac{2\cdot 4 \cdot \ldots \cdot 2n}{n^2 x^n} \right|$$

$$= \lim_{n\to\infty} |x| \cdot \underbrace{\frac{1}{2n+2}}_{\downarrow\, 0} \cdot \underbrace{\left(\frac{n+1}{n}\right)^2}_{\downarrow\, 1} = 0$$

So it always converges.

radius of conv $= \infty$

interval $= (-\infty, \infty)$.

11.9 # 4/6

$$\frac{5}{1-4x^2} = 5 \sum_{n=0}^{\infty} (4x^2)^n$$

$$= \sum_{n=0}^{\infty} 5\cdot 4^n x^{2n}$$

convergence: $|4x^2| < 1$

$|x^2| < 1/4$

$|x| < 1/2$

special cases:

$x = 1/2$: $\sum 5\cdot 4^n \left(\frac{1}{2}\right)^{2n} = \sum 5$ diverges

$x = -1/2$ $\sum 5\cdot 4^n \left(-1/2\right)^{2n} = \sum (-1)^n \cdot 5$ diverges

Interval of conv: $(-1/2, 1/2)$.

11.9 #8/10

$$\frac{x}{2x^2+1} = x \cdot \frac{1}{1-(-2x^2)}$$

$$= x \sum (-2x^2)^n$$

$$= x \sum (-1)^n \cdot 2^n \cdot x^{2n}$$

$$= \sum (-1)^n \cdot 2^n \cdot x^{2n+1}$$

Convergence: $|-2x^2| < 1$

$$|x^2| < \frac{1}{2}$$

$|x| < \frac{1}{\sqrt{2}}$ radius $= \frac{1}{\sqrt{2}}$