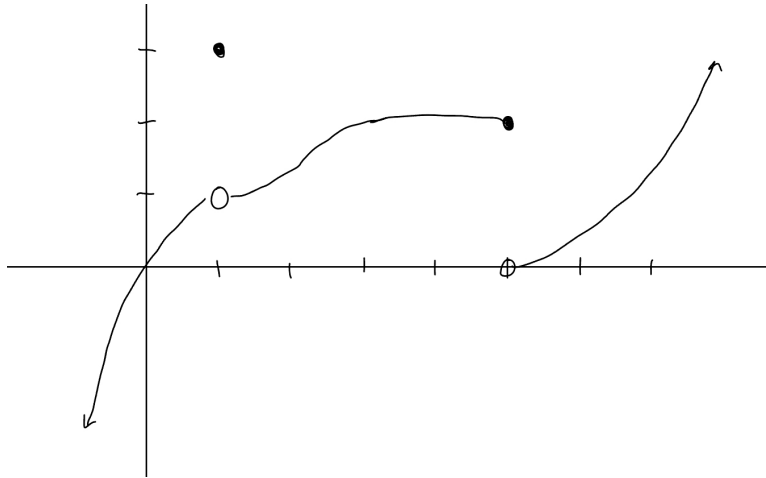


Math 1121 Exam #1

No calculators! You do not need to simplify numerical answers.
Submit your answers to gradescope in the usual way.

Question 1.



Above is a graph of $f(x)$. Please find the following limits and values, or say if they do not exist: (you must at least 7 right to get credit)

a) $f(1)$

3

e) $f(5)$

2

b) $\lim_{x \rightarrow 1} f(x)$

1

f) $\lim_{x \rightarrow 5^+} f(x)$

0

c) $\lim_{x \rightarrow 1^+} f(x)$

1

g) $\lim_{x \rightarrow 5^-} f(x)$

2

d) $\lim_{x \rightarrow 3} f(x)$

2

h) $\lim_{x \rightarrow 5} f(x)$

DNE

Question 2. Please find the limit:

$$\lim_{x \rightarrow -3} \frac{x^2 + 4x + 3}{x^2 - 5x - 24}$$

$$\lim_{x \rightarrow -3} \frac{\cancel{(x+3)}(x+1)}{(x-8)\cancel{(x+3)}}$$

$$= \lim_{x \rightarrow -3} \frac{x+1}{x-8} = \frac{-2}{-11} = \frac{2}{11}$$

Question 3. Please find the following limits: (you must get them all right to get credit)

a) $\lim_{x \rightarrow \infty} \frac{3x^2 - 4}{7x + x^3}$

0

b) $\lim_{x \rightarrow -\infty} \frac{4x}{1 - 9x}$

4/-9

c) $\lim_{x \rightarrow \infty} \frac{x^3 + 2x^2}{7x^2 - 3x + 1}$

DNE

d) $\lim_{x \rightarrow \infty} \frac{4x + 1}{4x + 7x^2}$

0

Question 4. For this function:

$$f(x) = \frac{x-3}{x^2-2x-3}$$

a) Please find any discontinuity points.

$$\begin{aligned} x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x-3 = 0 & \quad x+1 = 0 \\ \underline{x=3 \quad \quad x=-1} \end{aligned}$$

b) For each discontinuity point that you found in the first part, find the limit of $f(x)$ as x approaches that point.

$$\lim_{x \rightarrow 3} \frac{\cancel{x-3}}{(\cancel{x-3})(x+1)} = \lim_{x \rightarrow 3} \frac{1}{x+1} = \frac{1}{4}$$

$$\lim_{x \rightarrow -1} \frac{\cancel{x-3}}{(\cancel{x-3})(x+1)} = \lim_{x \rightarrow -1} \frac{1}{x+1} = \frac{1}{0} \text{ DNE}$$

Question 5. For this function:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq -1, \\ 4x + 6 & \text{if } -1 < x \leq 3, \\ 2x - 10 & \text{if } x > 3 \end{cases}$$

a) Please find any discontinuity points.

plugging $x = -1$: $(-1)^2 + 1 = 2$ so it is continuous at $x = -1$
 $4(-1) + 6 = 2$

$x = 3$: $4 \cdot 3 + 6 = 18$
 $2 \cdot 3 - 10 = -4$ $x = 3$ is a discontinuity

b) For each discontinuity point that you found in the first part, find the two one-sided limits of $f(x)$ as x approaches that point.

$$\lim_{x \rightarrow 3^-} f(x) = 18$$

$$\lim_{x \rightarrow 3^+} f(x) = -4$$

Question 6. For this function:

$$f(x) = 1 - x^2$$

please find the average rate of change from $x = -1$ to $x = 2$.

$$\begin{aligned} \frac{f(2) - f(-1)}{2 - (-1)} &= \frac{1 - 2^2 - (1 - (-1)^2)}{3} \\ &= \frac{1 - 4 - 0}{3} = \frac{-3}{3} = -1 \end{aligned}$$

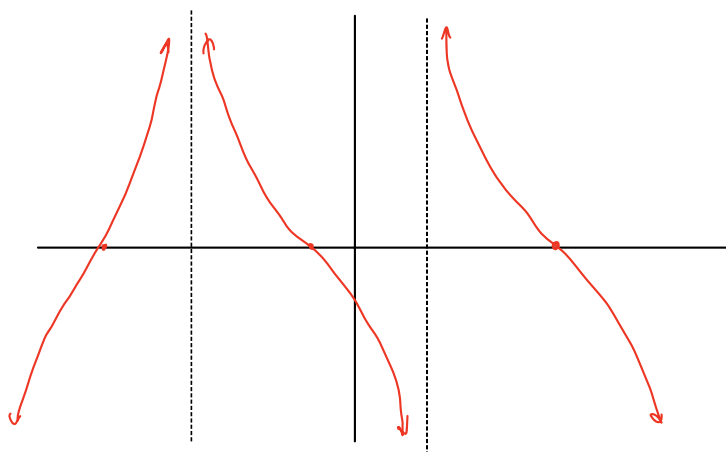
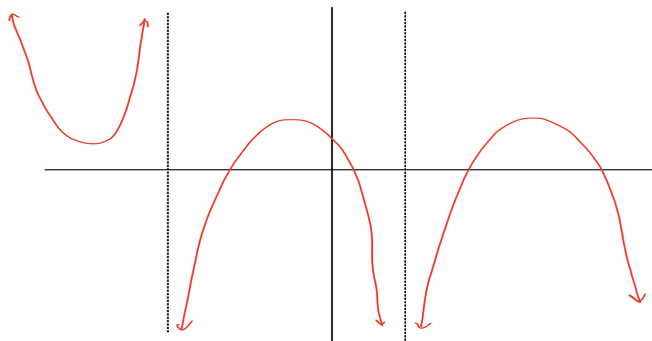
Question 7. Please use the definition of the derivative to find the instantaneous rate of change of this function

$$f(x) = x^2 - 2x + 1$$

at the point $x = 2$.

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(2+h)^2 - 2(2+h) + 1 - (2^2 - 2 \cdot 2 + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{4} + 4h + h^2 - \cancel{4} - 2h + \cancel{1} - (\cancel{4})}{h} = \lim_{h \rightarrow 0} \frac{4h + h^2 - 2h}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h + h^2}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}(2+h)}{\cancel{h}} = \lim_{h \rightarrow 0} 2+h = 2+0 = 2 \end{aligned}$$

Question 8. This is a graph of $f(x)$. Please sketch a graph of $f'(x)$.



Question 9. I have a side-hobby as a Z-list YouTuber (true story). My number of subscribers over time in thousands obeys this formula (made-up formula):

$$s(t) = t^2 + 2t$$

where t is the time in years since I started posting.

Please find $s'(5)$, and write a sentence explaining what this answer means. Be sure to include an explanation of what the number 5 refers to.

$$s'(t) = 2t + 2 \quad (\text{the derivative})$$

$$s'(5) = 2 \cdot 5 + 2 = 12$$

This means: At year 5, my # of subscribers is increasing at a rate of 12k per year.

Question 10. For each of the following, find the derivative: (you must get all 3 right to get credit)

a) $3x^2 + 8x - 1$

$$6x + 8$$

b) $6x^3 - x^{-4} + 7x$

$$18x^2 + 4x^{-5} + 7$$

c) $3x - 2 - x^{1/2}$

$$3 - \frac{1}{2}x^{-1/2}$$