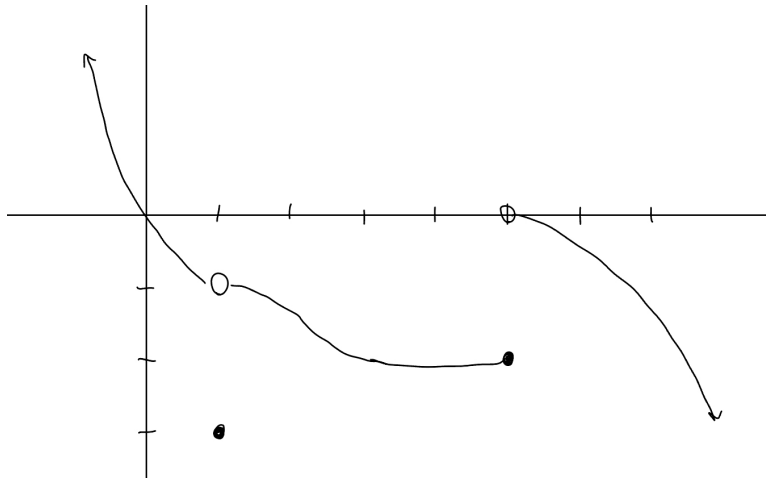# Math 1121 Exam #1

No calculators! You do not need to simplify numerical answers.
Submit your answers to gradescope in the usual way.

Question 1.

Above is a graph of $f(x)$. Please find the following limits and values, or say if they do not exist: (you must at least 7 right to get credit)

a) $f(1)$ -3

e) $f(5)$ -2

b) $\lim\limits_{x \to 1} f(x)$ -1

f) $\lim\limits_{x \to 5^+} f(x)$ 0

c) $\lim\limits_{x \to 1^+} f(x)$ -1

g) $\lim\limits_{x \to 5^-} f(x)$ -2

d) $\lim\limits_{x \to 3} f(x)$ -2

h) $\lim\limits_{x \to 5} f(x)$ DNE

Question 2. Please find the limit:

$$\lim_{x \to 3} \frac{(x-1)^2 - 4}{x - 3}$$

$$\frac{(3-1)^2 - 4}{3-3} = \frac{2^2 - 4}{0} = \frac{0}{0}$$

$$= \lim_{x \to 3} \frac{(x-1)(x-1) - 4}{x - 3} = \lim_{x \to 3} \frac{x^2 - 2x + 1 - 4}{x - 3} = \lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$$

$$= \lim_{x \to 3} \frac{(x-3)(x+1)}{x-3} = \lim_{x \to 3} x + 1 = 3 + 1 = 4$$

Question 3. Please find the following limits: (you must get them all right to get credit)

a) $\lim_{x \to \infty} \dfrac{3x^2 - 4}{7x + x^3}$ 0

b) $\lim_{x \to -\infty} \dfrac{4x}{1 - 9x}$ $4/-9$

c) $\lim_{x \to \infty} \dfrac{x^3 + 2x^2}{7x^2 - 3x + 1}$ DNE

d) $\lim_{x \to \infty} \dfrac{4x + 1}{4x + 7x^2}$ 0

Question 4. For this function:
$$f(x) = \frac{x^2 - 5x + 6}{x - 2}$$

a) Please find any discontinuity points.

$$\text{denom} = 0 \quad : \quad x = 2$$

b) For each discontinuity point that you found in the first part, find $f(x)$, and find the limit of $f(x)$ as x approaches that point.

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2} = \lim_{x \to 2} \frac{(x-3)(x-2)}{x-2} = \lim_{x \to 2} x - 3 = 2 - 3 = -1$$

Question 5. I have a dog named Furnando (not a true story). When I buy bags of dog food, they cost me $5 for the first bag, and then $4 each for any additional bags. Please write a piecewise function describing the cost of buying x bags.

$$f(x) = \begin{cases} 5 & \text{if } x = 1 \\ 5 + 4(x-1) & \text{if } x > 1 \end{cases}$$

Question 6. For this function:
$$f(x) = 1 - x^2$$
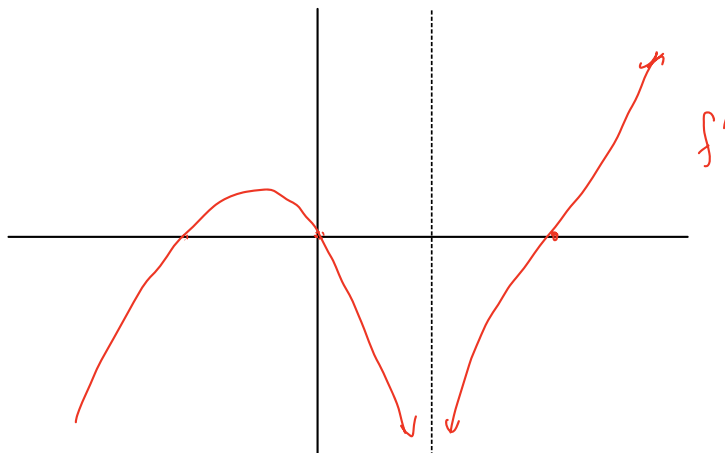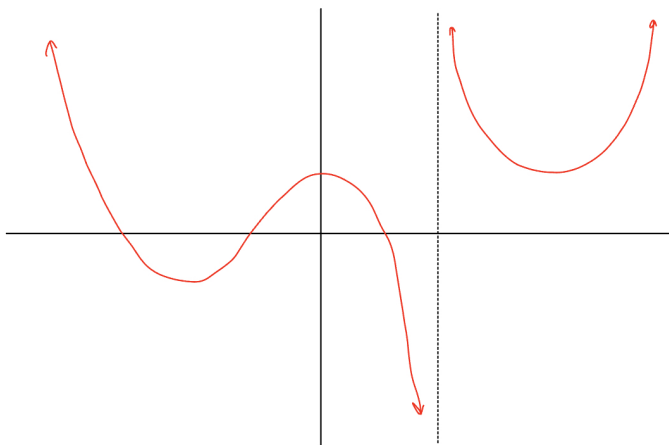please find the average rate of change from $x = -1$ to $x = 2$.

$$\frac{f(2) - f(-1)}{2 - -1} = \frac{1 - 2^2 - (1 - (-1)^2)}{3}$$

$$= \frac{1 - 4 - 0}{3} = \frac{-3}{3} = -1$$

Question 7. Please use the definition of the derivative to find the derivative of this function:
$$f(x) = 4x^2 + 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{4(x+h)^2 + 1 - (4x^2 + 1)}{h} = \lim_{h \to 0} \frac{4(x+h)(x+h) + 1 - 4x^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{4(x^2 + 2xh + h^2) - 4x^2}{h} = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 4x^2}{h}$$

$$= \lim_{h \to 0} \frac{8xh + 4h^2}{h} = \lim_{h \to 0} \frac{h(8x + 4h)}{h} = \lim_{h \to 0} 8x + 4h$$

$$= 8x + 4 \cdot 0 = 8x$$

Question 8. This is a graph of $f(x)$. Please sketch a graph of $f'(x)$.

f'

Question 9. My dog Furnando has fleas! The number of fleas living in Furnando's fur on day t is described by this function:

$$f(t) = 4t - t^2.$$

Please find $f'(10)$ and write a sentence explaining what this answer means. Be sure to include an explanation of what the number 10 refers to. Does your answer indicate good news or bad news for Furnando?

$$f'(t) = 4 - 2t \qquad \text{(the derivative)}$$

$$\text{so} \quad f'(10) = 4 - 2 \cdot 10 = 4 - 20 = -16$$

This means: on day 10, the # of fleas is decreasing by 16. Good news!

Question 10. For each of the following, find the derivative: (you must get all 3 right to get credit)

a) $3x^2 + 8x - 1$

$$6x + 8$$

b) $6x^3 - x^{-4} + 7x$

$$18x + 4x^{-5} + 7$$

c) $3x - 2 - x^{1/2}$

$$3 - \frac{1}{2}x^{-1/2}$$