

Math 1121 Exam #3 (new ones)

Question 21. Please give intervals where this function is concave up and concave down:

$$f(x) = x^4 + 8x^3 - 30x^2 + 12x + 2$$

$$f'(x) = 4x^3 + 24x^2 - 60x + 12$$

$$f''(x) = 12x^2 + 48x - 60$$

$$= 12(x^2 + 4x - 5)$$

$$= 12(x+5)(x-1)$$

$$x = -5, \quad x = 1$$



$$f''(-6) = 12 \cdot (-6+5)(-6-1) \text{ etc}$$
$$= + \quad - \quad -$$

$$\text{up: } (-\infty, -5) \cup (1, \infty)$$

$$\text{down: } (-5, 1)$$

Question 22. Please find and identify the relative extrema of $f(x)$ using the second derivative test:

$$f(x) = x^3 + 3x^2 - 45x + 2$$

$$f'(x) = 3x^2 + 6x - 45$$

$$= 3(x^2 + 2x - 15)$$

$$= 3(x+5)(x-3)$$

$$x = 3, -5$$

$$f''(x) = 6x + 6$$

$$f''(3) = 6 \cdot 3 + 6 = +$$

$$f''(-5) = 6 \cdot (-5) + 6 = -$$

3 is a relative min,

-5 - - - - max.

Question 23. Please use calculus techniques to sketch the graph of:

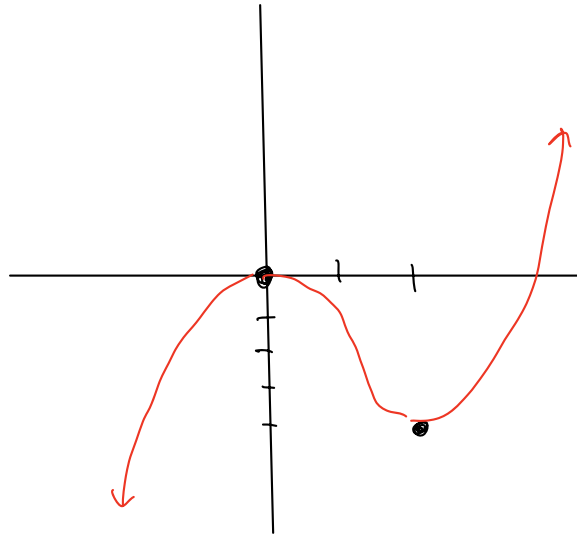
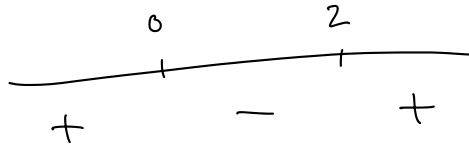
$$f(x) = x^3 - 3x^2$$

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ &= 3x(x - 2) \end{aligned}$$

crit #s: 0, 2

$$f(0) = 0^3 - 3 \cdot 0^2 = 0$$

$$\begin{aligned} f(2) &= 2^3 - 3 \cdot 2^2 \\ &= 8 - 3 \cdot 4 \\ &= 8 - 12 = -4 \end{aligned}$$



Question 24. Please find and identify the absolute extrema of

$$f(x) = x^3 - 3x + 1$$

on the interval $[0, 2]$.

$$f'(x) = 3x^2 - 3$$

$$3x^2 - 3 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

x	y
0	1
1	-1 ← min
2	3 ← max

Question 25. Please do the following integrals:

a) $\int 4x^2 - 2x + 1 dx$

$$\frac{4}{3}x^3 - x^2 + x + C$$

b) $\int 5x + \frac{4}{x^2} dx = \int 5x + 4x^{-2} dx$

$$\frac{5}{2}x^2 + 4 \cdot \frac{1}{-1}x^{-1} + C$$

c) $\int e^{3x} + 7^x dx$

$$\frac{1}{3}e^{3x} + \frac{1}{\ln 7} 7^x + C$$

Question 26. Please find a function $f(x)$ with $f'(x) = 6x^2 + 4x - 1$ and $f(1) = 5$.

$$\begin{aligned} f(x) &= \int 6x^2 + 4x - 1 \\ &= \frac{6}{3}x^3 + \frac{4}{2}x^2 - x + C \\ &= 2x^3 + 2x^2 - x + C \end{aligned}$$

$$5 = 2 \cdot 1^3 + 2 \cdot 1^2 - 1 + C$$

$$5 = 2 + 2 - 1 + C$$

$$5 = 3 + C$$

$$C = 2$$

$$f(x) = 2x^3 + 2x^2 - x + 2$$

Question 27. Please do the antiderivative:

$$\int x e^{5x^2+3} dx$$

$$u = 5x^2 + 3$$

$$du = 10x dx$$

$$\frac{1}{10} du = x dx$$

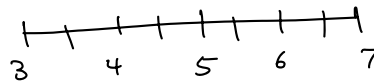
$$\int x e^u dx = \int e^u \cdot x dx = \int e^u \cdot \frac{1}{10} du = \frac{1}{10} \int e^u du$$

$$= \frac{1}{10} e^u + C = \frac{1}{10} e^{5x^2+3} + C$$

Question 28. Please estimate the definite integral:

$$\int_3^7 x - x^2 dx$$

using rectangles with $n = 8$ and...



a) the left endpoints.

$$\frac{1}{2} \left(f(3) + f(3.5) + \dots + f(6.5) \right) \\ = \frac{1}{2} \left(3 - 3^2 + 3.5 - 3.5^2 + \dots + 6.5 - 6.5^2 \right)$$

b) the right endpoints.

$$\frac{1}{2} \left(3.5 - 3.5^2 + 4 - 4^2 + \dots + 7 - 7^2 \right)$$

c) the midpoints.

$$\frac{1}{2} \left(3.25 - 3.25^2 + 3.75 - 3.75^2 + \dots + 6.75^2 - 6.75^2 \right)$$

Question 29. Please estimate the definite integral:

$$\int_3^7 4x - x^2 dx$$

using $n = 8$ and...

a) the trapezoid rule.

$$\frac{1}{2} \left(\frac{1}{2} (4 \cdot 3 - 3^2) + 4 \cdot 3.5 - 3.5^2 + \dots + 4 \cdot 6.5 - 6.5^2 + \frac{1}{2} (4 \cdot 7 - 7^2) \right)$$

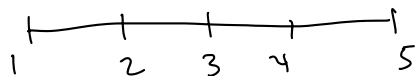
b) Simpson's rule.

$$\frac{1}{6} \left(4 \cdot 3 - 3^2 + 4(4 \cdot 3.5 - 3.5^2) + 2(4 \cdot 4 - 4^2) + \dots + 4(4 \cdot 6.5 - 6.5^2) + 4 \cdot 7 - 7^2 \right)$$

Question 30. I have an investment whose value changes day to day according to this formula:

$$f'(t) = .001e^{0.2t},$$

in dollars per day. Please estimate the total change in my investment's value from day 1 to day 5 using the trapezoid rule with $n = 4$.



$$\frac{5-1}{4} \left(\frac{1}{2} .001e^{.2 \cdot 1} + .001e^{.2 \cdot 2} + .001e^{.2 \cdot 3} + .001e^{.2 \cdot 4} + \frac{1}{2} .001e^{.2 \cdot 5} \right)$$

Question 31. Please evaluate these integrals:

a) $\int_0^3 x^2 - 4x \, dx$

$$\frac{1}{3}x^3 - 2x^2 \Big|_0^3 = \frac{1}{3} \cdot 3^3 - 2 \cdot 3^2 - \left(\frac{1}{3} \cdot 0^3 - 2 \cdot 0^2 \right)$$

b) $\int_1^4 \frac{4}{x} \, dx$

$$4 \ln x \Big|_1^4 = 4 \ln 4 - 4 \ln 1$$

c) $\int_2^8 4\sqrt{x} + e^x \, dx = \int_2^8 4x^{1/2} + e^x$

$$4 \cdot \frac{1}{3/2} x^{3/2} + e^x \Big|_2^8 = 4 \cdot \frac{2}{3} \cdot 8^{3/2} + e^8 - \left(4 \cdot \frac{2}{3} \cdot 2^{3/2} + e^2 \right)$$