Exam 3 old ones (11:00)

1. a. 3
   b. −1
   c. −1
   d. 0
   e. 1
   f. −1
   g. 1
   h. DNE

2. $\lim_{x\to 3} \frac{x^2-9}{x^2-3x} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{x\cancel{(x-3)}} = \frac{3+3}{3} = 2$

3. a) $4/2 = 2$

   b) DNE

   c) 0

   d) $3/{-1} = -3$

4. $f(x) = \frac{x^2-5x-14}{x^2+3x+2} = \frac{(x-7)(x+2)}{(x+1)(x+2)}$

   a) Discontinuities at $x = -1$ & $x = -2$

   b) $\lim_{x\to -1} \frac{(x-7)\cancel{(x+2)}}{(x+1)\cancel{(x+2)}} = \frac{-8}{0}$ $\boxed{\text{DNE}}$

   $\lim_{x\to -2} \frac{(x-7)(x+2)}{(x+1)(x+2)} = \frac{-9}{-1} = \boxed{9}$

5.

$$f(x) = \begin{cases} 3x - 4x^2 & \text{if} \quad x<0 \\ x^2+3x & \text{if} \quad 0 \le x \le 2 \\ 2x+10 & \text{if} \quad x>2 \end{cases}$$

a) plug 0: $3\cdot 0 - 4\cdot 0 = 0$ it is continuous at $x=0$

$0^2 + 3\cdot 0 = 0$

plug 2: $2^2 + 3\cdot 2 = 10$ $x=2$ is a discontinuity

$2\cdot 2 + 10 = 12$

b) $\lim_{x\to 2^-} f(x) = 10$

$\lim_{x\to 2^+} f(x) = 12$

6.

$$\frac{f(b)-f(a)}{b-a} = \frac{5-2\cdot 3^2+3 - (5-2\cdot 1^2+1)}{3-1}$$

$$= \frac{5-18+3-(5-2+1)}{2} = \frac{-10-4}{2} = \boxed{-7}$$

7.

$$f'(7) = \lim_{h\to 0} \frac{f(7+h)-f(7)}{h}$$

$49 - 21 + 1$

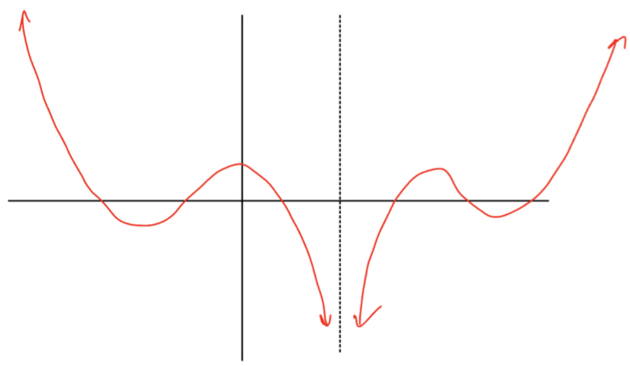
$$= \lim_{h\to 0} \frac{(7+h)^2 - 3(7+h)+1 - (7^2 - 3\cdot 7+1)}{h}$$

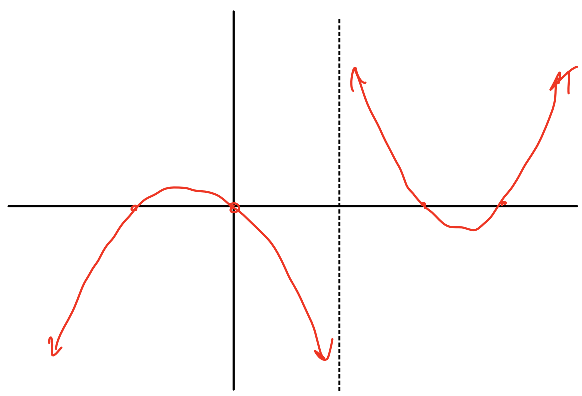
$$= \lim_{h\to 0} \frac{\cancel{49}+14h+h^2-\cancel{21}-3h\cancel{+1}-\cancel{29}}{h}$$

$$= \lim_{h\to 0} \frac{11h + h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(11+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 11 + h = \boxed{11}$$

8.





9 a) $a(4) = 20 - \sqrt{4} = 18$

After 4 minutes, the surface area is 18

b) $a'(t) = -\frac{1}{2} t^{-1/2}$

$$a'(4) = -\frac{1}{2} \cdot 4^{-1/2} = -\frac{1}{4} \cdot \frac{1}{\sqrt{4}} = -\frac{1}{8}$$

After 4 minutes, the surface area is decreasing by $1/8$.

10 a) $8x + 40x^4$

b) $5 \cdot \frac{1}{2} x^{-1/2}$

c) $\frac{d}{dx} 8x^{-3} + 10x^{-1} = -24x^{-4} - 10x^{-2}$

11. $5x^2(20x^4 - 21x^2 + 8) + (4x^5 - 7x^3 + 8x - 1)(10x)$

12. $$\frac{(x^8 - 3x)(10x - 7) - (5x^2 - 7x + 4)(8x^7 - 3)}{(x^8 - 3x)^2}$$

13. $\frac{1}{2}(4x^2 + 7x - 3)^{-1/2}(8x + 7)$

14. $(5x^2 - 3x) \cdot 4(x^3 + x^2 - 1)^3 (3x^2 + 2x) + (x^3 + x^2 - 1)^4 \cdot (10x - 3)$

15. a) $5^x \ln 5$

b) $9e^{8x^2+x}(16x+1)$

c) $5 \cdot 4^{2x} \cdot \ln 4 \cdot 2$

16. $2 \cdot 3^{4x} = 10$

$3^{4x} = 5$

$\ln 3^{4x} = \ln 5$

$4x \ln 3 = \ln 5$

$x = \frac{\ln 5}{4 \ln 3}$

17. a) $\frac{1}{x\ln 4}$

b) $\frac{1}{x^4+2x-1} \cdot (4x^3+2)$

c) $3 \cdot \frac{1}{(5x^2+9x)\ln 7} \cdot (10x+9)$

18. $18x \cdot 10^{x^2+x} \ln 10 \cdot (2x+1) + 10^{x^2+x} \cdot 18$

19. $f(x) = x^4 - 8x^2 + 4$

$f'(x) = 4x^3 - 16x$

$= 4x(x^2-4)$

$= 4x(x+2)(x-2)$

$x = 0, 2, -2$

$-$	-2	$+$	0	$-$	2	$+$

$f'(-3) = 4(-3)(-3+2)(-3-2)$
$+ \ - \ - \ -$

$f'(-1) = 4(-1)(-1+2)(-1-2)$
$+ \ - \ + \ -$

$f'(1) = 4 \cdot 1 \cdot (1+2)(1-2)$
$+ \ + \ + \ -$

$f'(3) = + \ + \ + \ +$

increasing: $(-2,0)$ & $(2,\infty)$

decreasing: $(-\infty,-2)$ & $(0,2)$

20. $f(x) = \dfrac{5-x}{2+x}$

$$f'(x) = \frac{(2+x)\cdot(-1) - (5-x)\cdot 1}{(2+x)^2}$$

$$= \frac{-2 \cancel{-x} - 5 \cancel{+x}}{(2+x)^2} = \frac{-7}{(2+x)^2}$$

$f' = 0$ never

f' DNE : $x = -2$

		-2	
f'	$-$	DNE	$-$

$f'(x) = \dfrac{-7}{(\ )^2} = \dfrac{-}{+}$ always $-$

decreasing $(-\infty, -2)$ & $(-2, \infty)$