Math 1121 Exam #3 (new ones)

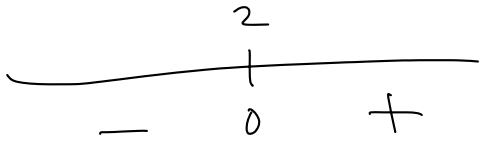
Question 21. Please give intervals where this function is concave up and concave down:

$$f(x) = x^3 - 6x^2 + 2x - 3$$

$f'(x) = 3x^2 - 12x + 2$

$f''(x) = 6x - 12 = 6(x-2)$

$f'' = 0$: $x = 2$



$f''(0) = 6(0-2)$

$f''(3) = 6(3-2)$

Concave down: $(-\infty, 2)$

up: $(2, \infty)$

Question 22. Please find and identify the relative extrema of $f(x)$ using the second derivative test:

$$f(x) = x^3 - 6x^2 + 2$$

$f'(x) = 3x^2 - 12x$

$= 3x(x-4)$

Critical #s: $x = 0$, $x = 4$

$f''(x) = 6x - 12$

$f''(0) = -12$ — So $x=0$ is a relative max

$f''(4) = 6 \cdot 4 - 12$ + So $x = 4$ is a relative min.

Question 23. Please sketch the graph of:

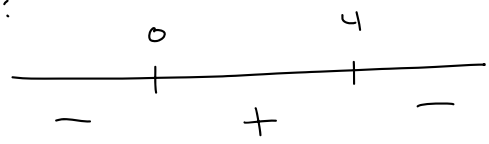
$$f(x) = 2x^2 - \frac{1}{3}x^3$$

$f'(x) = 4x - x^2$

$f'(x) = x(4-x)$

critical #s: $0, 4$

inc/dec:



$f'(-1) = (-1)(4-(-1))$

$f'(1) = 1(4-1)$

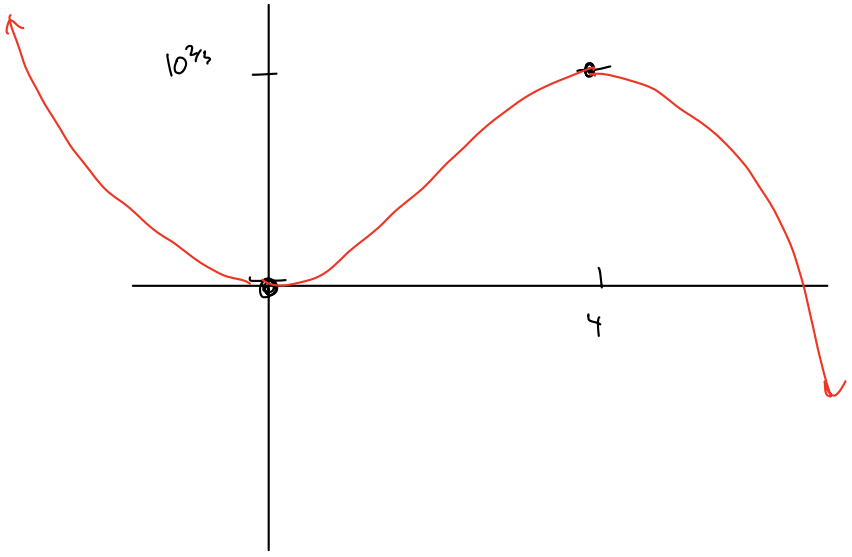
$f'(5) = 5(4-5)$

y-values:

$x=0$: $y = f(0) = 2\cdot 0^2 - \frac{1}{3}\cdot 0^3 = 0$

$x=4$: $y = f(4) = 2\cdot 4^2 - \frac{1}{3}\cdot 4^3$

$= 32 - \frac{1}{3}\cdot 64 = \frac{96}{3} - \frac{64}{3} = \frac{32}{3} = 10\frac{2}{3}$



Question 24. Please find and identify the absolute extrema of

$$f(x) = x^3 - 3x$$

on the interval $[0, 2]$.

$f'(x) = 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3(x+1)(x-1)$

crit #s: 1, -1

x	y
0	$0^3 - 3 \cdot 0 = 0$
2	$2^3 - 3 \cdot 2 = 2$ ← max
1	$1^3 - 3 \cdot 1 = -2$ ← min
~~-1~~	

outside interval

$x = 1$, $y = -2$ is the abs min,

$x = 2$, $y = 2$ is the abs max.

Question 25. Please do the following integrals:

a) $\int 4x^2 - 2x + 1\, dx$

$\frac{4}{3}x^3 - x^2 + x + C$

b) $\int x^2(x+1)\, dx$

$\int x^3 + x^2\, dx$

$= \frac{1}{4}x^4 + \frac{1}{3}x^3 + C$

c) $\int e^{3x} + x^3\, dx$

$\frac{1}{3}e^{3x} + \frac{1}{4}x^4 + C$

Question 26. Please find a function $f(x)$ with $f'(x) = 3x^2 + 8x - 3$ and $f(1) = 5$.

$$f(x) = \int 3x^2 + 8x - 3\,dx = x^3 + 4x^2 - 3x + C$$

$$5 = 1^3 + 4 \cdot 1^2 - 3 \cdot 1 + C$$

$$5 = 1 + 4 - 3 + C$$

$$5 = 2 + C \qquad C = 3$$

$$\boxed{f(x) = x^3 + 4x^2 - 3x + 3}$$

Question 27. Please do the antiderivative:

$$\int x^2\sqrt{5 - 2x^3}\,dx$$

$$u = 5 - 2x^3$$

$$du = -6x^2\,dx$$

$$-\frac{1}{6}du = x^2\,dx$$

$$\int x^2 u^{1/2}\,dx = \int u^{1/2} \cdot x^2\,dx$$

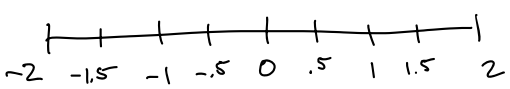
$$= \int u^{1/2} \cdot -\frac{1}{6}\,du = \frac{-1}{6}\int u^{1/2}\,du$$

$$= -\frac{1}{6} \cdot \frac{2}{3}u^{3/2} + C$$

$$= -\frac{1}{6} \cdot \frac{2}{3}(5 - 2x^3)^{3/2} + C$$

Question 28. Please estimate the area under the curve of

$$f(x) = x^2 + e^x$$



$\Delta x = .5$

from $x = -2$ to $x = 2$ using rectangles with $n = 8$ and...

a) the left endpoints.

$$.5\left(f(-2) + f(-1.5) + \ldots + f(1.5)\right)$$

$$= .5\left((-2)^2 + e^{-2} + (-1.5)^2 + e^{-1.5} + \ldots + 1.5^2 + e^{1.5}\right)$$

b) the right endpoints.

$$.5\left[(-1.5)^2 + e^{-1.5} + (-1)^2 + e^{-1} + \ldots + 2^2 + e^2\right)$$

c) the midpoints.

$$.5\left((-1.75)^2 + e^{-1.75} + (-1.25)^2 + e^{-1.25} + \ldots + 1.75^2 + e^{1.75}\right)$$

Question 29. Please estimate the area under the curve of

$$f(x) = x^2 + e^x$$

from $x = -2$ to $x = 2$ using $n = 8$ and...

a) the trapezoid rule.

$$\frac{2-(-2)}{8}\left(\frac{1}{2}\left((-2)^2 + e^{-2}\right) + (-1.5)^2 + e^{-1.5} + \ldots + 1.5^2 + e^{1.5} + \frac{1}{2}\left(2^2 + e^2\right)\right)$$

b) Simpson's rule.

$$\frac{2-(-2)}{3\cdot 8}\left((-2)^2 + e^{-2} + 4\left((-1.5)^2 + e^{-1.5}\right) + 2\left((-1)^2 + e^{-1}\right) + \ldots + 4\left(1.5^2 + e^{1.5}\right) + 2^2 + e^2\right)$$

Question 30. I have an investment whose value changes day to day according to this formula:

$$f'(t) = .004t + .01,$$

in dollars per day. Please find the total change in my investment's value from day 1 to day 5.

$$\int_1^5 .004t + .01\,dt = .002t^2 + .01t \Big|_1^5$$

$$= .002 \cdot 5^2 - .01 \cdot 5 - \left(.002 \cdot 1^2 - .01 \cdot 1\right)$$

Question 31. Please evaluate these integrals:

a) $\int_0^3 4x - 9x^2 + 1\,dx$

$$2x^2 - 3x^3 + x \Big|_0^3 = 2 \cdot 3^2 - 3 \cdot 3^3 + 3 - \left(2 \cdot 0^2 - 3 \cdot 0^3 + 0\right)$$

b) $\int_1^4 \frac{4}{x}\,dx$

$$4\ln|x| \Big|_1^4 = 4\ln 4 - 4\ln 1$$

c) $\int_2^8 4\sqrt{x} + e^x\,dx = \int_2^8 4x^{1/2} + e^x$

$$4 \cdot \frac{1}{3/2} x^{3/2} + e^x \Big|_2^8 = 4 \cdot \frac{2}{3} \cdot 8^{3/2} + e^8 - \left(4 \cdot \frac{2}{3} \cdot 2^{3/2} + e^2\right)$$