Math 1121 exam 2 practice problems

1 Limits on graphs

1. Find some limit by looking at a graph (like book problems 3.1 #5-12)

2 Limits algebraically

2. Compute: $\lim_{x\to 1} \dfrac{x^2+2x+1}{x^2-1}$

3 Limits at ∞

3. Compute: $\lim_{x\to\infty} \dfrac{7x^2+2x+1}{1-x^2}$
4. Compute: $\lim_{x\to-\infty} \dfrac{x}{x^2-1}$

4 Continuity

5. Find any discontinuities of $\frac{x+1}{x^2-4}$, and find the limit of $f(x)$ as it approaches each discontinuity.

5 Piecewise functions

6. Find any discontinuities of

$$f(x) = \begin{cases} x^2-3 & \text{if } x < 1 \\ x+3 & \text{if } 1 \le x < 5 \\ 8 & \text{if } x \ge 5 \end{cases}$$

   For each discontinuity, find the left and right one-sided limits as you approach that point.

7. When I pay my taxes, I have to pay 15% of any income below \$100,000, and 25% of any further income. (These are made up numbers.) Write a piecewise function that describes how much taxes I would owe if my income is x dollars.

6 Average rate of change

8. Find the average rate of change of $2x^2-4x$ from $x=3$ to $x=6$
9. Find the average rate of change from $x=-1$ to $x=4$ of the piecewise function called $f(x)$ above.

7 Definition of the derivative

10. Use the definition of the derivative to find the instantaneous rate of change of $f(x)=2x^2-x$ at $x=3$.
11. Use the definition of the derivative to find $f'(0)$, for $f(x)=4x^3+2x-3$.
12. Use the definition of the derivative to find the derivative of either of these.

8 Derivatives on graphs

13. Look at a graph of $f(x)$, and you draw $f'(x)$. (like book problems 11th ed. 3.5 #7 and 8, or 12th ed 3.5 #15 and 16)

9 Derivatives in word problems

14. My blood pressure is going up! From day to day, my blood pressure is given by the formula $p(t) = 72 + .1t + .02t^2$, where t is the time in days. Find $p(10)$ and $p'(10)$, and explain in a few words what each one means. (The numbers for this one are slightly weird- you can use a calculator if you like. Test questions will have less weird numbers.)

10 Derivatives of polynomials (power rule)

15. Find the derivative of $5\sqrt{x} - 6/x^5 + x^2/3$

16. Find x where the slope of $f(x) = x^3 + 15x^2 + 73x - 10$ is 1.

11 Product rule

17. Find the derivative of $f(x) = (4x^7 - 2x^3 + 4)(3x^2 - 1)$

12 Quotient rule

18. Find the derivative of $f(x) = \frac{4x^7 - 2x^3 + 4}{3x^2 - 1}$

13 Chain rule

19. For $f(x) = (4x^2 - 3)^4$, find $f'(1)$

20. Find the derivative of $\sqrt{x^5 - 2x}$

14 Derivatives misc (rational functions)

("Rational functions" means some combination of polynomials.)

21. Find the derivative of $f(x) = 4x(x^2 - 4x + 3)^2$

22. Find the derivative of $f(x) = \frac{(2x+3)(3x+2)}{8x - 7x^2}$

23. Find the derivative of $p(y) = \big((5x^2 - 7x)(8x^{10} + 1)\big)^4$

15 Exponential functions and their derivatives

24. Find the derivative of e^{4x^2}

25. Find the derivative of $7^{x^2 - 3}$

16 Logarithms basics

26. Compute by hand: $\log_4 16, \log_3 \frac{1}{27}, \log \sqrt{10}, \ln 1$
27. Solve for x in $3^x + 2 = 35$ (on the test, leave your answer unsimplified as something in terms of ln)

17 Derivatives of logs

28. Find the derivative of $5^x + \log_4 x - x^2$
29. Find the derivative of $6 \log_4(5 - 4x)$

18 Derivatives misc (exp & logs)

30. Find the derivative of $(\ln(4x^2 - 3))^6$
31. Find the derivative of $4^{x^2 - x} \log_3 x$
32. Find the derivative of $\frac{2^x}{(\log_7 x)^3}$

19 Increasing & decreasing (polynomials)

33. Give intervals where $x^3 - 9x^2 - 21x - 10$ is increasing and decreasing.

20 Increasing & decreasing (harder)

34. Give intervals where $\frac{x+2}{x-3}$ is increasing and decreasing.
35. Give intervals where xe^{2x+3} is increasing and decreasing.

Things to memorize

- Trick for doing $\lim_{x \to \infty}$ of a fraction of polynomials
- Average rate of change formula
- Definition of the derivative
- Product, quotient, chain rules
- Derivative of exponentials
- Derivative of logs

Answers!

2. This limit does not exist.
3. -7
4. 0
5. $x = 2$ and $x = -2$ are discontinuities. The limits $\lim_{x\to 2} f(x)$ and $\lim_{x\to -2} f(x)$ both do not exist.
6. $x = 1$ is a discontinuity. $\lim_{x\to 1^-} f(x) = -2$, and $\lim_{x\to 1^+} f(x) = 4$.
7. $f(x) = \begin{cases} 0.15x & \text{if } x < 100,000 \\ 15,000 + 0.25(x - 100,000) & \text{if } x \geq 100,000 \end{cases}$
8. $\frac{2\cdot 6^2 - 4\cdot 6 - (2\cdot 3^2 - 4\cdot 3)}{6-3} = 14$
9. $\frac{4+3-((-1)^2-3)}{4-(-1)} = \frac{9}{5}$
10. 11
11. 2
12. They are $4x - 1$ and $12x^2 + 2$. (Using the definition of the derivative, these are each difficult to do. The second is probably too hard to put on the test.)
14. $p(10) = 73.2$ (just plug in $t = 10$). This means that my blood pressure on day 10 is 73.2. For $p'(10)$, take the derivative and then plug in $t = 10$. You get $p'(10) = 0.5$. This means that on day 10, my blood pressure is increasing at a rate of 0.5 per day.
15. $2.5x^{-1/2} + 30x^{-6} + \frac{2}{3}x$
16. $x = -4$ and $x = -6$
17. $(4x^7 - 2x^3 + 4)(6x) + (3x^2 - 1)(28x^6 - 6x^2)$
18. $\frac{(3x^2-1)(28x^6-6x^2)-(4x^7-2x^3+4)(6x)}{(3x^2-1)^2}$
19. $f'(x) = 4(4x^2 - 3)^3(8x)$, so $f'(1) = 32$.
20. $\frac{1}{2}(x^5 - 2x)^{-1/2}(5x^4 - 2)$
21. $4x \cdot 2(x^2 - 4x + 3)(2x - 4) + (x^2 - 4x + 3)^2 \cdot 4$
22. $\frac{(8x-7x^2)((2x+3)(3)-(3x+2)(2))-(2x+3)(3x+2)(8-14x)}{(8x-7x^2)^2}$
23. $4((5x^2 - 7x)(8x^{10} + 1))^3((5x^2 - 7x)(80x^9) + (8x^{10} + 1)(10x - 7))$
24. $e^{4x^2}(8x)$
25. $7^{x^2-3}(\ln 7)(2x)$
26. $2, -3, 1/2, 0$
27. $\frac{\ln 33}{\ln 3}$
28. $5^x \ln 5 + \frac{1}{x \ln 4} - 2x$

29. $6\frac{1}{(5-4x)\ln 4}(-4)$

30. $6(\ln(4x^2-3))^5\frac{1}{4x^2-3}(8x)$

31. $4^{x^2-x}\frac{1}{x\ln 3}+\log_3 x\cdot 4^{x^2-x}\ln 4\cdot(2x-1)$

32. $\frac{(\log_7 x)^3 2^x \ln 2 - 2^x\cdot 3(\log_7 x)^2\frac{1}{x\ln 7}}{(\log_7 x)^6}$

33. Increasing on $(-\infty,-1)$ and $(7,\infty)$, decreasing on $(-1,7)$.

34. Decreasing on $(-\infty,3)$ and $(3,\infty)$, never increasing.

35. Decreasing on $(-\infty,-1/2)$, increasing on $(-1/2,\infty)$.