# Math 1121 exam 3 practice problems

## 21 Concavity

1. For $f(x) = x^3 - 9x^2 + 3x - 2$, give intervals where $f$ is concave up and concave down.

## 22 Second derivative test

2. Use the second derivative test to find and identify all relative extrema of $2x^3 - 18x^2 - 6$

## 23 Curve sketching

3. Sketch the graph of $x^4 - 4x^3$

## 24 Absolute extrema

4. Find the absolute extrema of $2x^3 - 18x^2 - 6$ on $[-1, 1]$.

## 25 The indefinite integral (antiderivatives)

5. $\displaystyle\int \frac{1}{x^2} + \frac{1}{x} - 2x^3\,dx$

6. $\displaystyle\int 4 \cdot 7^{2x} + \frac{8}{x}\,dx$

7. $\displaystyle\int x^2(4x - 7)\,dx$

## 26 Initial value problems

8. Find some function with $f'(x) = 6x^2 - x + 2$ and $f(2) = 3$.

9. My profits are changing at a rate of $p'(t) = e^{2t} - 3$. My profit at time 0 is 15. Find a formula for my profit with no arbitrary constants.

## 27 $u$-substitution

10. $\displaystyle\int e^x \sqrt{7 + 2e^x}\,dx$

11. $\displaystyle\int \frac{x}{(x^2 - 3)^3}\,dx$

### 28 Definite integral estimation by rectangles

12. Estimate $\int_0^3 2x - 7\,dx$ using $n = 6$, with rectangles on the left endpoints.

13. Estimate the area under $y = x^2$ between $x = 1$ and $x = 3$ using $n = 4$ with rectangles on the midpoints.

### 29 Trapezoid & Simpson's rule

14. Estimate $\int_0^3 2x - 7\,dx$ using $n = 6$, with the trapezoid rule.

15. Estimate the area under $y = x^2$ between $x = 1$ and $x = 3$ using $n = 4$ with Simpson's rule.

### 30 Total change in word problems

16. My dog Woofenstein is running down the hallway for 10 seconds. I measure his velocity in m/s at each second as follows:

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $v(t)$ | 2 | 2.7 | 3.2 | 3.5 | 3.3 | 3.4 | 3.4 | 3.2 | 2.1 | 1.2 | 0.2 |

Estimate the total distance he traveled between $t = 2$ and $t = 8$ using Simpson's rule with $n = 6$.

17. Let's say that my merch store profits are changing according to this formula

$$P'(t) = 35 - 10t,$$

where $t$ is measured in months. Give the total change in profits from month 2 to month 5. (Here I didn't say "estimate", so you should use the fundamental theorem of calculus.)

### 31 Fundamental theorem of calculus

18. $\int_1^5 x^2 + 3x - 7\,dx$

19. $\int_0^3 4^{4x} + 2x\,dx$

## Formulas I will give you on the test

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} e^x = e^x \qquad \int e^x\,dx = e^x + C$$

$$\frac{d}{dx} a^x = a^x \ln a \qquad \int a^x\,dx = \frac{1}{\ln a} a^x + C$$

$$\int e^{kx}\,dx = \frac{1}{k} e^{kx} + C \qquad \int a^{kx}\,dx = \frac{1}{k \ln a} a^{kx} + C$$

$$\frac{d}{dx} \ln x = \frac{1}{x} \qquad \frac{d}{dx} \log_a x = \frac{1}{x \ln a}$$

$$\frac{b-a}{n} \left( \frac{1}{2} f(x_0) + f(x_1) + \cdots + f(x_{n-1}) + \frac{1}{2} f(x_n) \right)$$

$$\frac{b-a}{3n} \left( f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n) \right)$$

## Answers!

1. Concave down on $(-\infty, 3)$ and concave up on $(3, \infty)$.
2. $x = 0$ is a relative maximum, $x = 6$ is a relative minimum.
3. (you can check the graph on your calculator)
4. Absolute minimum at $x = -1$ (with $y = -26$) and absolute maximum at $x = 0$ (and $y = -6$).
5. $-x^{-1} + \ln|x| - \frac{1}{2}x^4 + C$
6. $\frac{4}{2\ln 7}7^{2x} + 8\ln|x| + C$
7. $x^4 - \frac{7}{3}x^3 + C$
8. $f(x) = 2x^3 - \frac{1}{2}x^2 + 2x - 15$
9. $p(t) = \frac{1}{2}e^{2t} - 3t + 14.5$
10. $\frac{1}{3}(7 + 2e^x)^{3/2} + C$
11. $-\frac{1}{4}(x^2 - 3)^{-2} + C$
12. $0.5(2 \cdot 0 - 7 + 2 \cdot .5 - 7 + \cdots + 2 \cdot 2.5 - 7)$
13. $0.5(1.25^2 + 1.75^2 + 2.25^2 + 2.75^2)$
14. $\frac{1}{2}(\frac{1}{2}(2 \cdot 0 - 7) + 2 \cdot .5 - 7 + \cdots + 2 \cdot 2.5 - 7 + \frac{1}{2}(2 \cdot 3 - 7))$
15. $\frac{1}{6}(1^2 + 4 \cdot 1.5^2 + 2 \cdot 2^2 + 4 \cdot 2.5^2 + 3^2)$
16. $\frac{1}{3}(3.2 + 4 \cdot 3.5 + 2 \cdot 3.3 + 4 \cdot 3.4 + 2 \cdot 3.4 + 4 \cdot 3.2 + 2.1)$
17. We are doing $\int_2^5 35 - 10t\,dt$, and the answer is $35 \cdot 5 - 5 \cdot 5^2 - (35 \cdot 2 - 5 \cdot 2^2)$
18. $\frac{1}{3} \cdot 5^3 + \frac{3}{2} \cdot 5^2 - 7 \cdot 5 - (\frac{1}{3} \cdot 1^3 + \frac{3}{2} \cdot 1^2 - 7 \cdot 1)$
19. $\frac{1}{4\ln 4}4^{12} + 9 - \frac{1}{4\ln 4}$