Math 3342 Exam #1

Question 1. (4 points each) In each part, please give a regular expression which generates the language described. In all parts, you should assume the alphabet is $\Sigma = \{a, b\}$.

a) All strings which start and end with a, and use only b in between.

$$a\, b^* \, a$$

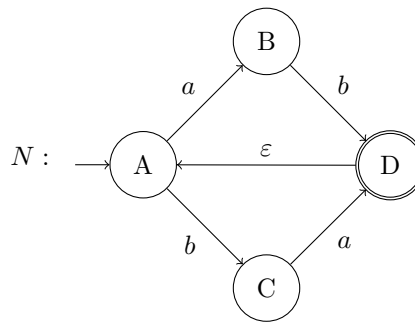b) All strings in which every b is followed immediately by an a.

$$(ba + a)^*$$

c) All strings which start and end with the same letter.

$$a(a+b)^* a + b(a+b)^* b$$

d) All strings with even length.

$$\left((a+b)(a+b)\right)^* \quad \text{or} \quad \left(aa + ab + ba + bb\right)^*$$

Question 2. This whole page is about this NFA:

a) (5 points) Please give the formal description of this N.

$$N = (\{A,B,C,D\}, \{a,b\}, \delta, A, \{D\})$$

where:

$\delta(A,a) = \{B\}$ $\delta(B,a) = \emptyset$ $\delta(C,a) = \{D\}$ $\delta(D,a) = \emptyset$

$\delta(A,b) = \{C\}$ $\delta(B,b) = \{D\}$ $\delta(C,b) = \emptyset$ $\delta(D,b) = \emptyset$

$\delta(A,\varepsilon) = \emptyset$ $\delta(B,\varepsilon) = \emptyset$ $\delta(C,\varepsilon) = \emptyset$ $\delta(D,\varepsilon) = \{A\}$

b) (5 points) Please write in ordinary words what it means to say $B \in \delta^*(C, aaba)$. Is this statement true?

It means if we start in state C and do $aaba$, we can end up in state B.   This is true.

c) (5 points) Please describe $L(N)$, either in words or using set theory notation.

All strings made up of 1 or more blocks of the form (ab) or (ba).   (Not all blocks need to be the same)

d) (4 points) Is $L(N)$ a regular language? Please say briefly why or why not.

Yes — "regular language" means its the language of an NFA or DFA, which it is.

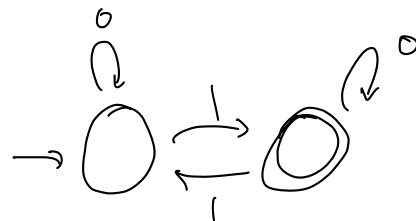**Question 3.** (15 points) For the same NFA N, please use the subset construction to make an equivalent DFA.
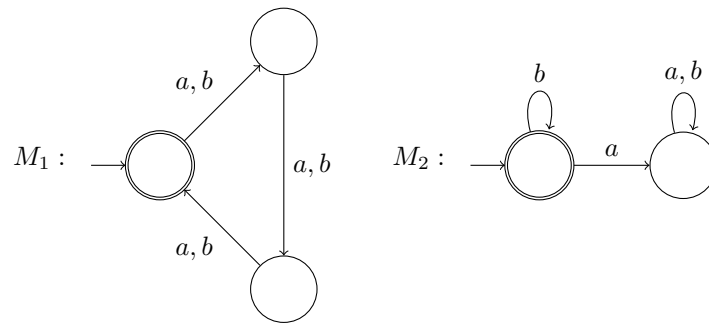
Here is the NFA again:
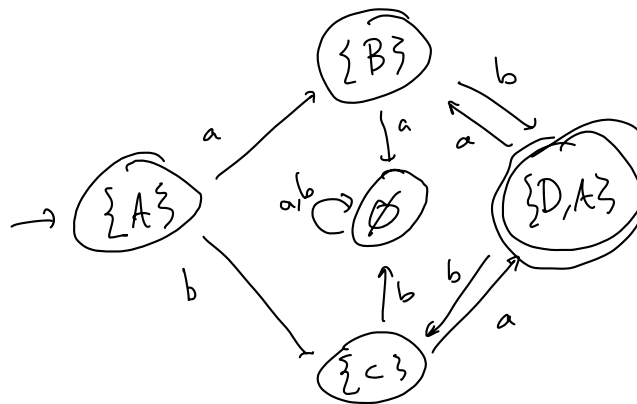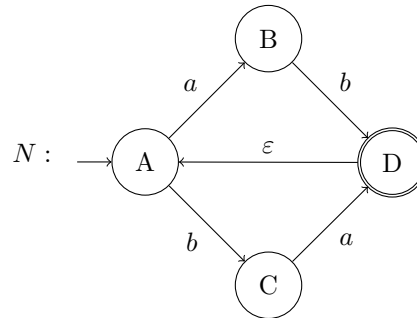
Question 4. (15 points) Please give a NFA for the set of all strings on $\Sigma = \{a, b\}$ which include either aab or baa (or both) as a substring.
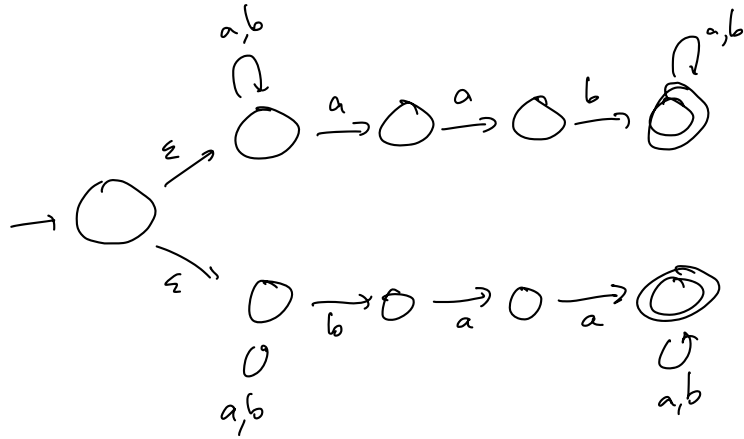
Question 5. (15 points) Please give a DFA for the set of all binary strings which use an odd number of the digit 1.

Question 6. This whole page is about these two DFAs:

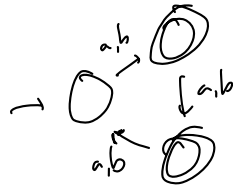M_1 : a, b ... a, b ... a, b

M_2 : b a, b a

a) (5 points) Please describe $L(M_1)$ and $L(M_2)$ is ordinary words or using set theory notation.

$$L(M_1) = \{x \mid \text{length of } |x| \text{ is a multiple of } 3\}$$

$$L(M_2) = \{x \mid x \text{ uses no } a\text{'s}\}$$

b) (5 points) Please give a DFA for $\overline{L(M_1)}$, where the bar means complement.

Same as M_1, but swap accepting/rejecting states

c) (10 points) Please give a DFA for $L(M_1) \cup L(M_2)$.