Name: ____________________________

Math 3342 Exam #2

Question 1. This whole page is about this stack machine:

read	pop	push
a	S	SXX
b	X	ε
ε	S	ε

a) Please find some nonempty string that is accepted by the stack machine, and write a derivation on the stack machine showing that it is accepted.

$(abb, S) \mapsto (bb, SXX) \mapsto (bb, XX)$

$\mapsto (b, X) \mapsto (\varepsilon, \varepsilon)$ accepted!

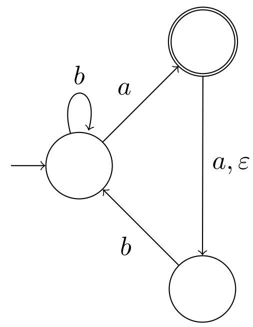
b) Please find some nonempty string that is rejected by the stack machine, and write a derivation on the stack machine showing that it is rejected.

$(b, S) \mapsto (b, \varepsilon)$ stuck!

c) What is the language of this stack machine? You can describe the language in words, or using set-theory notation.

$\{ a^n b^{2n} \}$

Question 2. Both parts are about this NFA:



a) Please give a regular expression which is equivalent to this NFA.

$b^* a((a+\varepsilon) b b^* a)^*$ or $(b^* a (a+\varepsilon) b)^* a$

b) Please give a grammar which is equivalent to this NFA.

$S \to bS \mid aT$
$T \to aR \mid R \mid \varepsilon$
$R \to bS$

c) Choose a string of length more than 1 that is accepted on the NFA, and show a grammar derivation for that string.

aaba:

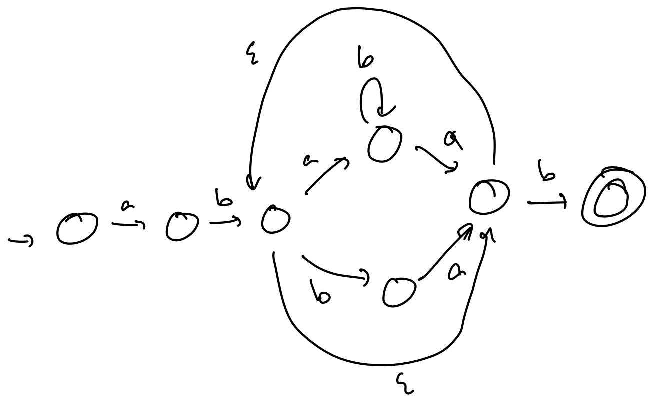
$S \to aT \to aaR \to aabS \to aabaT \to aaba$ ✓

d) Is your grammar above context-free? Say briefly why. (Say enough so that I know that you know what context-free means.)

Yes! Context-free just means the left side of each arrow is a single nonterminal letter, which is true in this case.

Question 3. Make an NFA that is equivalent to this regular expression:

$$ab(ab^*a + ba)^*b$$



Question 4. Please show that this language is nonregular:

$$L = \{a^n x b^n \mid n \in \mathbb{N}, x \in \{a, b\}^*\}$$

Let $D_i = \frac{\delta}{\delta a} L = \{ a^{n-i} x b^n \}$

These are all different, so L is nonregular.

Question 5. In these 4 parts, please make a grammar for the given language, or say that it's impossible.

a) $\{a^n b c^m\}$

$$S \to AbC$$
$$A \to aA \mid \varepsilon$$
$$C \to cC \mid \varepsilon$$

b) $\{a^n b c^n\}$

$$S \to aSc \mid b$$

c) $\{x c a^n b^m \mid x \in \{a, b, c\}^*\}$

$$S \to XcAB$$
$$X \to aX \mid bX \mid cX \mid \varepsilon$$
$$A \to aA \mid \varepsilon$$
$$B \to bB \mid \varepsilon$$

d) Please choose one of the grammars that you made above, and write an equivalent stack machine.

b)

read	pop	push
ε	S	aSc
ε	S	b
a	a	ε
b	b	ε
c	c	ε

Question 6. Please make a grammar for all strings which look like whole numbers using commas to separate blocks of 3 digits. So your grammar should be able to generate things like:

10,000 125 2 42,003,190

but NOT things like

4021 42,32 5,32,8 4,

(For simplicity, let's allow things with 0 as the leftmost digit, so it's OK if your grammar can generate something like "000,000".)

$S \to T \mid S,DDD$

$T \to D \mid DD \mid DDD$

$D \to 0 \mid 1 \mid 2 \mid \ldots \mid 9$

Question 7. Here is a wrong proof that $L = \{a^n b^m\}$ is not a regular language:

> **Proof:** Let $D_i = \frac{d}{da^i} L = \{a^{n-i} b^m\}$. These are all different for different i, since for example a^{n-1} is different from a^{n-2}. Thus L has infinitely many different derivatives, so it is not regular.

What exactly is the error in the wrong proof above? Explain using perhaps a couple sentences.

n is a variable which could be any number.

So n-1 also could just be any number,

So n-2 also could just be any number. So these aren't different.

So $\{a^{n-i} b^m\}$ are actually all the same for various i.