Homework #10

Each problem describes a function. Show that the function is Turing computable by demonstrating a Turing machine that computes it.

Question 1. The "head" function, which takes a nonempty string on the alphabet $\{a, b\}^*$ and returns just its first letter.

Question 2. The "tail" function, which takes a nonempty string on the alphabet $\{a, b\}^*$ and returns the whole thing except the first letter.

Question 3. For $x \in \{a, b\}^*$, the function $f(x) = a^{|x|}$.

Question 4. Consider only strings using the letter a, and let $f(a^n) = a^{2n}$. (One possible strategy you could use: first turn the string a^n into something like $\bar{a}^n b^n$ by marking each a and writing b at the end to match it. Then run across the whole string and turn everything into an a, resulting in a^{2n}.)

Question 5. Consider only even-length strings using the letter a, and let $f(a^{2n}) = a^n$. (You can use a similar strategy: mark an a at the begining, and then blank one out at the end. Then when you're done, change every $\bar{a}$ back to an a.)

Question 6. For binary numbers, $f(x) = 4x$.

Question 7. For binary numbers, $f(x) = x + 4$. (You can assume that x is large enough so that it already uses at least 3 digits.)

Question 8. For binary numbers, $f(x) = x + 5$. (Again assume that x uses at least 3 digits. Hint: First add 1, then add 4.)

Question 9. For binary numbers, $f(x) = x \div 4$. (Give only the quotient as the answer, ignoring any remainders.)

Question 10. For binary numbers, $f(x) = x \mod 4$.