Name: _____

Math 1015: Exam #1

Question 1. Please find the winner using plurality. Show enough work so that I can tell what you're doing.

4	2	3	1	3
A	B	D	C	B
B	C	B	A	A
C	A	A	D	D
D	D	C	B	C

A: 4
B: 5
D: 3
C: 1

B wins

Question 2. Please find the winner using Ranked choice voting. Show enough work so that I can tell what you're doing.

4	2	3	1	3
A	B	D	C	B
B	C	B	A	A
C	A	A	D	D
D	D	C	B	C

Rd 1

A: 4
B: 5
~~C: 1~~
D: 3

Rd 2

4	2	3	1	3
A	B	D	A	B
B	A	B	D	A
D	D	A	B	D

A: 5
B: 5
~~D: 3~~

Rd 3

4	2	3	1	3
A	B	B	A	B
B	A	A	B	A

A: 5
B: 8

B wins

Question 3. Please find the winner using Condorcet's method, or say that there is no winner.

4	2	3
A	C	B
C	B	C
B	A	A

A vs B: A: 4, B: 5

A vs C: A: 4, C: 5

B vs C: B: 3, C: 6

C wins

Question 4. Please find the winner using the Borda count.

4	2	3	2	
A	B	C	B	2
B	C	B	A	1
C	A	A	C	0

A: $4 \cdot 2 + 0 + 0 + 2 \cdot 1 = 10$

B: $4 + 4 + 3 + 4 = 15$

C: $0 + 2 + 6 + 0 = 8$

B wins!

Question 5. Please explain why Condorcet's method satisfies the majority criterion.

Imagine X is ranked 1st by a majority of voters.

Then X will win in all their 1v1 comparisons,

So X will be the winner with Cond.'s method.

Question 6. Use this sample election to show how some of the voters can manipulate the election if we're using Borda. Write some words explaining why your example qualifies as a manipulation.

6	3	2	
A	C	B	2
C	A	C	1
B	B	A	0

A: $6 \cdot 2 + 3 \cdot 1 = 15$

B: 4

C: $6 + 3 \cdot 2 + 2 = 14$

A wins.

B C A can change to C B A. Then it looks like

6	3	2	
A	C	C	2
C	A	B	1
B	B	A	0

A: $6 \cdot 2 + 3 \cdot 1 = 15$

B: $0 + 0 + 2 = 2$

C: $6 + 3 \cdot 2 + 2 \cdot 2 = 16$

now C wins, which is an improved result.

Question 7. Consider this election using the random dictator method. Please give the probability for each candidate to win. (Write your probabilities as fractions– you don't need to convert to percentages.)

3	2	4	2
A	B	C	C
B	A	B	A
C	C	A	B

→ total = 11

A: 3/11

B: 2/11

C: 6/11

Question 8. Please translate this ranked election into an approval voting election, assuming that each voter approves of their top 2 choices. Show the chart of approval ballots that would result, and determine the winner using approval voting.

3	4	2
A	B	C
D	C	A
C	A	D
B	D	B

	3	4	2
A	X		X
B		X	
C		X	X
D	X		

A: 5

B: 4

C: 6

D: 3

C wins

Question 9. Please consider the weighted voting system: $[15 : 7, 6, 3, 2, 2, 1]$

For each part, your answer in a few words. (You won't need to say much, but you must say more than yes or no.)

a) Identify any dictators, or say that there are none.

None - no weight makes it to 15 by itself.

b) Does the 7 have veto power?

Yes! The others together make $6+3+2+2+1 = 14$, which doesn't meet the quota. So the 7 is necessary to meet the quota.

c) Does the 6 have veto power?

No! You can do $7+3+2+2+1 = 15$, so you don't need the 6 to make the quota.

d) Is the 1 a dummy?

No! The 1 can be important, like in $7+3+2+2+1 = 15$.

So the 1 can influence the result, so it's not a dummy.

e) Is each 2 a dummy?

No! Similar to above, the 2's are important, for example in $7+3+2+2+1 = 15$.

Question 10. Please find the Shapley-Shubik power index for this weighted voting system:

A B C

$[20 : 15, 12, 7]$

perms	weights	pivotal
A B C	15, (12), 7	B
A C B	15, (7), 12	C
B A C	12, (15), 7	A
B C A	12, 7, (15)	A
C A B	7, (15), 12	A
C B A	7, (15), 12	A

A: 4/6

B: 1/6

C: 1/6