# Math 1015: Homework #3

**Question 1.** Here are the votes for the 2022 special election in Alaska between Mary Peltola, Sarah Palin, and Nick Begich (rounded, in thousands):

| 48 | 34 | 27 | 24 | 21 | 16 | 11 | 5 | 4 |
|-----|-----|-----|-----|-----|-----|----|-----|-----|
| Pel | Pal | B | Pel | Pal | B | B | Pel | Pal |
| B | B | Pal | | | Pel | | Pal | Pel |
| Pal | Pel | Pel | | | Pal | | B | B |

Please find the winner using the Borda count. (In any columns where some candidates do not appear, those candidates should get 0 points.)

**Question 2.** Explain why Ranked Choice Voting satisfies the Majority Winner criterion.

**Question 3.** Explain why the random dictator method does not satisfy the Majority Winner criterion.

**Question 4.** Here is a new criterion:
   **Majority Loser criterion**: *if some candidate is ranked last by a majority of the voters, then that candidate should lose the election.*

a) Please explain why the Condorcet method does satisfy the majority loser criterion.

b) Please invent an example showing the the plurality system does not satisfy the majority loser criterion. (Your example should have one candidate who is ranked last by a majority of the voters, but that candidate wins the election.)

**Question 5.** Consider this election, using plurality:

| 2 | 4 | 3 |
|---|---|---|
| A | B | D |
| C | A | A |
| B | D | C |
| D | C | B |

a) Describe how the DACB voters can manipulate this election.

b) Explain why the ACBD voters cannot manipulate this election.

c) If we use Borda instead of plurality, can any voters manipulate the election? Say specifically how somebody could manipulate, or explain why it's impossible.

**Question 6.** I'm voting with my friends about the best local dog salon. Our choices are "Snip Doggy Dog" (D), "Sham Pooch" (P), "Woof" (W), and "Ruff Cut" (R). (These are all real places.) People's votes are like this:

| 1 | 2 | 1 | 1 | 4 |
|---|---|---|---|---|
| R | W | D | D | P |
| D | R | R | R | W |
| W | D | P | W | D |
| P | P | W | P | R |

Please find the probabilities of each salon winning if we use the random dictator method.

**Question 7.** In June 2025 there was a democratic primary election for New York City Mayor using Ranked Choice Voting. There were many candidates on the ballot, but the top 3 candidates were Andrew Cuomo (C), Brad Lander (L), and Zohran Mamdani (M). The voters rankings for just these three candidates were as follows (in thousands):

| 375 | 282 | 85 | 83 | 82 | 47 | 35 | 31 | 29 |
|-----|-----|----|----|----|----|----|----|----|
| M | C | L | M | C | C | L | L | M |
| L |   | M |   | L | M |   | C | C |

Please find the probabilities for each candidate to win if we use the random dictator method.

**Question 8.** Invent and describe a situation where it might be a good idea to use the random dictator method, and also a situation where it would be a bad idea to use the random dictator method.

**Question 9.** Find the results of this approval election:

|   | 3 | 2 | 2 | 1 | 1 |
|---|---|---|---|---|---|
| A | X |   |   | X | X |
| B | X | X |   |   | X |
| C |   | X | X |   |   |
| D |   | X | X |   |   |

**Question 10.** In the context of approval voting, the Majority Winner criterion would say: *If a majority of voters approve of candidate X, then X will win the election.* Is that really true for approval voting? Say why or why not.