Math 1172 Midterm
New ones (everybody do these)

You do not need to simplify your answers, except if I say so.

Question 12. Please do the integral:

$$\int \sin^3 x \cos^2 x \, dx$$

$$\int \sin^2 x \cos^2 x \quad \sin x dx \qquad u = \cos x$$
$$du = -\sin x dx$$

$$\int (1-\cos^2 x) \cos^2 x \quad \sin x dx = -\int (1-u^2) u^2 du$$

$$= -\int u^2 - u^4 \, du = -\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right) + C$$

$$= -\left(\frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x\right) + C$$

Question 13. Please do the integral:

$$\int_0^1 x^2\sqrt{x^2+1}\,dx$$

$x = \tan\theta$ $\quad x = 0, \quad \theta = 0$

$dx = \sec^2\theta\, d\theta$ $\quad x = 1, \quad \theta = \pi/4$

$$\int_0^{\pi/4} \tan^2\theta \sqrt{\tan^2\theta + 1} \quad \sec^2\theta\, d\theta$$

$$= \int_0^{\pi/4} \frac{\sin^2\theta}{\cos^2\theta} \cdot \sqrt{\sec^2\theta} \quad \sec^2\theta \; d\theta \quad = \int_0^{\pi/4} \frac{\sin^2\theta}{\cos^5\theta} \quad d\theta$$

Question 14. Please do the integral:

$$\int \frac{5x+22}{x^2+x-20}\,dx \qquad = \int \frac{5x+22}{(x+5)(x-4)}$$

$$\frac{5x+22}{(x+5)(x-4)} = \frac{A}{x+5} + \frac{B}{x-4}$$

$$Ax - 4A + Bx + 5B = 5x + 22$$

$$A + B = 5 \qquad\qquad A = B - 5$$

$$-4A + 5B = 22$$

$$-4(5-B) + 5B = 22$$

$$-20 + 4B + 5B = 22$$

$$A+B=5 \rightarrow$$

$$9B = 42$$

$$A = 5 - 14/3 = 1/3$$

$$B = 42/9 = 14/3$$

So $$\int \frac{5x+22}{x^2+x-20} = \int \frac{1/3}{x+5} + \frac{14/3}{x-4}$$

$$= \frac{1}{3}\ln|x+5| + 14/3\,\ln|x-4|$$

Old ones
(only do the ones which you don't already have full credit on)

I am not leaving enough space for you to write your answers here. Use separate paper!

Question 1. a) Please find the derivative of $f(x) = \frac{5x}{\sin^2 x}$

b) Please find the antiderivative of $f(x) = 5x^2 - \frac{3}{x^4}$

Question 2. Please find the integral:

$$\int_1^4 \frac{x^2}{\sqrt{4 - x^3}}\, dx$$

Question 3. Please find the area bounded between the curves $y = x + 1$ and $y = x^2 - 1$.

Question 4. Please use the washer method to find the volume obtained when revolving the area between the curves $y = x$ and $y = x^2$ around the y-axis.

Question 5. Please use the cylindrical shells method to find the volume obtained when revolving the area between the curves $y = x$ and $y = x^2$ around the line $x = -1$.

Question 6. a) For $f(x) = x^2 \ln x$, please find $f'(1)$ and simplify as much as you can.

b) Please do the integral $\displaystyle\int_0^e \frac{x}{1 + x^2}\, dx$ and simplify as much as you can.

Question 7. a) Please find the derivative of $\cos(e^{3x^2+x})$

b) Please find the integral $\displaystyle\int_0^\pi \sin x e^{5+\cos x}\, dx$ and simplify as much as you can.

Question 8. a) Please do the integral $\displaystyle\int 4x6^{x^2}\, dx$

b) In each part find the logarithm by hand, or say "impossible" if it's not possible to do it by hand:

i) $\log_2 16$

ii) $\log 100$

iii) $\log_4 8$

iv) $\log_9 \frac{1}{3}$

Question 9. Please do the integral and simplify as much as you can:

$$\int_1^2 \frac{1}{\sqrt{4 - x^2}}\, dx$$

Question 10. Please find the limit: $\displaystyle\lim_{x \to 2} \frac{\ln(x - 1)}{x^2 - 4}$

Question 11. Please do the integral:

$$\int_0^1 (2 - x^2)e^{4x}\, dx$$

#1

a. $$\frac{\sin^2 x \cdot 5 - 5x \cdot 2\sin x\cos x}{\sin^4 x}$$

b. $$\int 5x^2 - 3x^{-4}\,dx = \frac{5}{3}x^3 + x^{-3} + C$$

#2

$$\int_1^4 \frac{x^2}{\sqrt{4-x^3}}\,dx$$

$u = 4 - x^3$

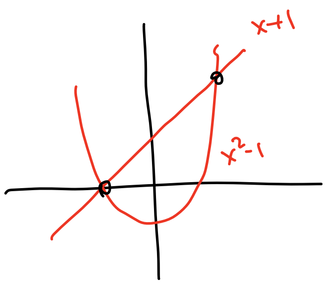
$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$$= \int \frac{1}{\sqrt{u}} \cdot -\frac{1}{3}du = -\frac{1}{3}\int u^{-1/2}\,du = \frac{-1}{3} \cdot 2u^{1/2}$$

$$= -\frac{2}{3}(4-x^3)^{1/2}\Big|_1^4 = \frac{-2}{3}\cdot(4-4^3)^{1/2} - \frac{-2}{3}(3)^{1/2}$$

#3



$x+1 = x^2-1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

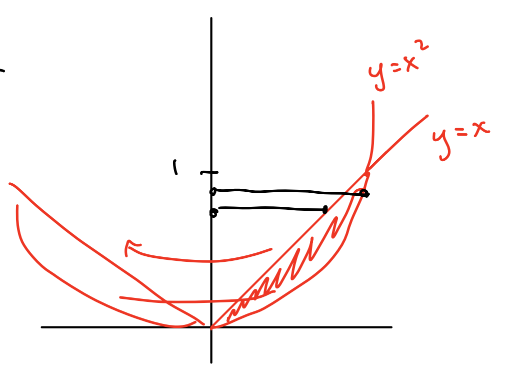
$x-2 = 0 \qquad x+1 = 0$

$x = 2 \qquad x = -1$

$$\int_{-1}^{2} x+1-(x^2-1)\,dx = \int_{-1}^{2} x - x^2 + 2\,dx$$

$$= \frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\Big|_{-1}^{2} = \frac{1}{2}\cdot 2^2 - \frac{1}{3}\cdot 2^3 + 2\cdot 2 - \left(\frac{1}{2}\cdot(-1)^2 - \frac{1}{3}(-1)^3 + 2(-1)\right)$$

#4



$x = y^{1/2}$

$x = y$

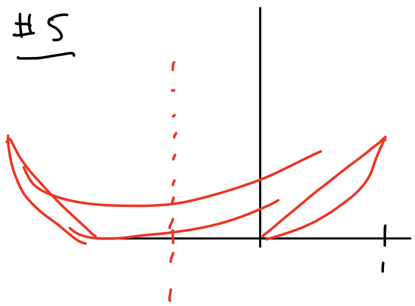
inner $= y$

outer $= y^{1/2}$

$$V = \pi \int_0^1 (y^{1/2})^2 - y^2\, dy = \pi \int_0^1 y - y^2\, dy$$

$$= \pi \left(\tfrac{1}{2} y^2 - \tfrac{1}{3} y^3 \right) \Big|_0^1 = \pi \left(\tfrac{1}{2} - \tfrac{1}{3} \right)$$

#5



$$2\pi \int_0^1 (x+1)(x-x^2)\, dx$$

$$= 2\pi \int_0^1 \cancel{x^2} + x - x^3 - \cancel{x^2}\, dx$$

$$= 2\pi \left(\tfrac{1}{2} x^2 - \tfrac{1}{4} x^4 \right) \Big|_0^1 = 2\pi \left(\tfrac{1}{2} - \tfrac{1}{4} \right) = \pi$$

#6

a

$$f'(x) = x^2 \cdot \tfrac{1}{x} + \ln x \cdot 2x$$

$$= x + 2x \ln x$$

$$f'(1) = 1 + 2 \cdot 1 \cdot \ln 1 = 1 + 2 \cdot 0 = \boxed{1}$$

b

$$\int_0^e \frac{x}{1+x^2}\, dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$\tfrac{1}{2} du = x\,dx$

$$\int \frac{1}{u} \cdot \tfrac{1}{2} du = \tfrac{1}{2} \ln|u| = \tfrac{1}{2} \ln|1+x^2| \Big|_0^e$$

$$= \tfrac{1}{2} \ln|1+e^2| - \tfrac{1}{2} \ln|1+0^2| = \tfrac{1}{2} \ln|1+e^2|$$

#7

a

$$-\sin(e^{3x^2+x}) \cdot e^{3x^2+x} \cdot (6x+1)$$

b

$$\int_0^\pi \sin x \; e^{5+\cos x}\, dx$$

$u = 5 + \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$$= -\int e^u \, du = -e^u = -e^{5+\cos x} \Big|_0^{\pi}$$

$$= -e^{5+\cos\pi} - -e^{5+\cos 0}$$

$$= -e^4 + e^6$$

#8

a $\int 4x\, 6^{x^2}\, dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$$= 4\int 6^u \cdot \frac{1}{2} du = 2\int 6^u \, du = 2 \cdot \frac{1}{\ln 6} 6^u + C$$

$$= 2 \cdot \frac{1}{\ln 6} 6^{x^2} + C$$

b

$\log_2 16 = 4$

$\log 100 = 2$

$\log_4 40$ is impossible by hand

$\log_9 \frac{1}{3} = -\frac{1}{2}$

9

$$\int_1^2 \frac{1}{\sqrt{4-x^2}} dx = \int_1^2 \frac{1}{\sqrt{4}\sqrt{1-\frac{x^2}{4}}} dx$$

$$= \frac{1}{2}\int_1^2 \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} dx$$

$u = x/2$

$du = \frac{1}{2} dx$

$2\,du = dx$

$$= \frac{1}{\cancel{2}}\int \frac{1}{\sqrt{1-u^2}} \cdot \cancel{2}\, du = \arcsin u$$

$$= \arcsin \frac{x}{2} \Big|_1^2 = \arcsin \frac{2}{2} - \arcsin \frac{1}{2}$$

$$= \arcsin 1 - \arcsin \frac{1}{2} = \pi/2 - \pi/6$$

10

$$\lim_{x\to 2} \frac{\ln(x-1)}{x^2-4} \to \frac{\ln(2-1)}{2^2-4} = \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 2} \frac{\frac{1}{x-1}}{2x} = \lim_{x\to 2} \frac{1}{2x(x-1)} = \frac{1}{2\cdot 2\cdot(2-1)} = \frac{1}{4}$$

4

$$\int (2-x^2)\, e^{4x}\, dx$$

$u = 2-x^2$ $\quad du = -2x\, dx$

$dv = e^{4x} dx$ $\quad v = \frac{1}{4} e^{4x}$

$$= (2-x^2)\cdot \frac{1}{4} e^{4x} - \int \frac{1}{4} e^{4x} \cdot -2x\, dx$$

$u = x$ $\quad du = dx$

$dv = e^{4x} dx$ $\quad v = \frac{1}{4} e^{4x}$

$$= (2-x^2)\cdot \frac{1}{4} e^{4x} + \frac{1}{2} \int x e^{4x}\, dx$$

$$= (2-x^2)\cdot \frac{1}{4} e^{4x} + \frac{1}{2}\left(x\cdot \frac{1}{4} e^{4x} - \int \frac{1}{4} e^{4x}\, dx \right)$$

$$= (2-x^2)\cdot \frac{1}{4} e^{4x} + \frac{1}{2}\left(x\cdot \frac{1}{4} e^{4x} - \frac{1}{4}\cdot \frac{1}{4} e^{4x} \right) + C$$