Math 1172 Midterm
New ones (everybody do these)

You do not need to simplify your answers, except if I say so.

Question 12. Please do the integral:

$$\int \sin^2 x \cos^2 x \, dx$$

$$\int \frac{1}{2}(1-\cos 2x)\, \frac{1}{2}(1+\cos 2x)\, dx$$

$$\frac{1}{4}\int (1-\cos 2x)(1+\cos 2x)\, dx$$

$$\frac{1}{4}\int 1 - \cos^2 2x \, dx$$

$$\frac{1}{4}\int 1 - \frac{1}{2}(1+\cos 4x)\, dx$$

$$\frac{1}{4}\int 1 - \frac{1}{2} - \frac{1}{2}\cos 4x \, dx$$

$$\frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos 4x \, dx$$

$$\frac{1}{4}\left(\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{4}\sin 4x\right) + C$$

Question 13. Please do the integral:

$$\int_0^1 x^3\sqrt{1-x^2}\,dx$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$x=1: \quad \sin\theta = 1$

$\theta = \pi/2$

$x=0: \quad \sin\theta = 0$

$\theta = 0$

$$\int_0^{\pi/2} \sin^3\theta \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} \sin^3\theta \sqrt{\cos^2\theta}\,\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} \sin^3\theta \cos^2\theta\, d\theta$$

$$= \int_0^{\pi/2} \sin^2\theta \cos^2\theta \sin\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= \int_0^{\pi/2} (1-\cos^2\theta)\cos^2\theta \sin\theta\, d\theta$$

$$= -\int (1-u^2)\,u^2\, du = -\int u^2 - u^4\, du$$

$$= -\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right) = -\left(\frac{1}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta\right)\Big|_0^{\pi/2}$$

$$= -\left(\frac{1}{3}\cos^3\pi/2 - \frac{1}{5}\cos^5\pi/2\right) - -\left(\frac{1}{3}\cos^3 0 - \frac{1}{5}\cos^5 0\right)$$

$$= -\left(0 - 0\right) + \left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= \frac{1}{3} - \frac{1}{5} = 2/15$$

Question 14. Please do the integral:

$$\int \frac{2x^2+x-2}{x^2+x}\,dx$$

$$x^2+x \,\overline{)\,2x^2+x-2\,}^{\;2}$$
$$\underline{2x^2+2x}$$
$$-x-2$$

$$= \int 2 + \frac{-x-2}{x^2+x} \, dx$$

$$\frac{-x-2}{x^2+x} = \frac{-x-2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)}$$

$$Ax + A + Bx = -x-2$$

$$(A+B)x + A = -x-2$$

$$A+B = -1 \longrightarrow -2+B = -1$$
$$A = -2 \qquad\qquad B = 1$$

$$\int 2 + \frac{-2}{x} + \frac{1}{x+1} \, dx$$

$$= 2x - 2\ln|x| + \ln|x+1| + C$$

Old ones
(only do the ones which you don't already have full credit on)

I am not leaving enough space for you to write your answers here. Use separate paper!

Question 1. Please do the integral and simplify all trig functions in your answer:

$$\int_{\pi/6}^{3\pi/2} \frac{3}{\sqrt{x}} + \sin x \, dx$$

Question 2. Please find the integral:

$$\int_1^4 \frac{x^2}{\sqrt{4 - x^3}} \, dx$$

Question 3. Please find the area bounded between the curves $y = x + 1$ and $y = x^2 - 1$.

Question 4. Please use the washer method to find the volume obtained when revolving the area between the curves $y = x$ and $y = x^2$ around the y-axis.

Question 5. Please use the cylindrical shells method to find the volume obtained when revolving the area between the curves $y = x$ and $y = x^2$ around the line $x = -1$.

Question 6. a) For $f(x) = x^2 \ln x$, please find $f'(1)$ and simplify as much as you can.

b) Please do the integral $\displaystyle\int_0^e \frac{x}{1 + x^2} \, dx$ and simplify as much as you can.

Question 7. a) Please find the derivative of $\cos(e^{3x^2+x})$

b) Please find the integral $\displaystyle\int_0^\pi \sin x e^{5+\cos x} \, dx$ and simplify as much as you can.

Question 8. a) Please do the integral $\displaystyle\int 4x6^{x^2} \, dx$

b) In each part find the logarithm by hand, or say "impossible" if it's not possible to do it by hand:

i) $\log_2 16$

ii) $\log 100$

iii) $\log_4 8$

iv) $\log_9 \frac{1}{3}$

Question 9. Please do the integral and simplify as much as you can:

$$\int_1^2 \frac{1}{\sqrt{4 - x^2}} \, dx$$

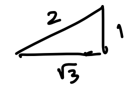
Question 10. Please find the limit: $\displaystyle\lim_{x \to 2} \frac{\ln(x - 1)}{x^2 - 4}$

Question 11. Please do the integral:

$$\int_0^1 (2 - x^2) e^{4x} \, dx$$

1

$$\int_{\pi/6}^{3\pi/2} \frac{3}{\sqrt{x}} + \sin x \, dx = \int_{\pi/6}^{3\pi/2} 3x^{-1/2} + \sin x \, dx$$



$$= 6x^{1/2} - \cos x \Big|_{\pi/6}^{3\pi/2} = 6\cdot\left(\tfrac{3\pi}{2}\right)^{1/2} - \cos\tfrac{3\pi}{2} - \left(6\cdot\left(\tfrac{\pi}{6}\right)^{1/2} - \cos\tfrac{\pi}{6}\right)$$

$$= 6\cdot\left(\tfrac{3\pi}{2}\right)^{1/2} - 0 - \left(6\left(\tfrac{\pi}{6}\right)^{1/2} - \tfrac{\sqrt{3}}{2}\right)$$

#2

$$\int_1^4 \frac{x^2}{\sqrt{4-x^3}}\,dx$$

$u = 4 - x^3$

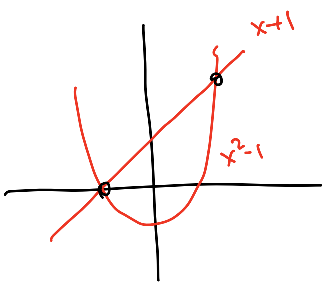
$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$$= \int \frac{1}{\sqrt{u}} \cdot -\frac{1}{3}du = -\frac{1}{3}\int u^{-1/2}\,du = \frac{-1}{3}\cdot 2u^{1/2}$$

$$= -\frac{2}{3}(4-x^3)^{1/2}\Big|_1^4 = \frac{-2}{3}\cdot(4-4^3)^{1/2} - \frac{-2}{3}(3)^{1/2}$$

#3



$x+1 = x^2-1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

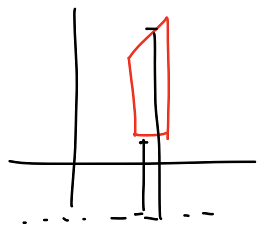
$x-2 = 0 \qquad x+1 = 0$

$x = 2 \qquad x = -1$

$$\int_{-1}^{2} x+1-(x^2-1)\,dx = \int_{-1}^{2} x - x^2 + 2\,dx$$

$$= \frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\Big|_{-1}^{2} = \frac{1}{2}\cdot 2^2 - \frac{1}{3}\cdot 2^3 + 2\cdot 2 - \left(\frac{1}{2}\cdot(-1)^2 - \frac{1}{3}(-1)^3 + 2(-1)\right)$$

4



inner = 2

outer = $\frac{x}{2}+2-(-1)$

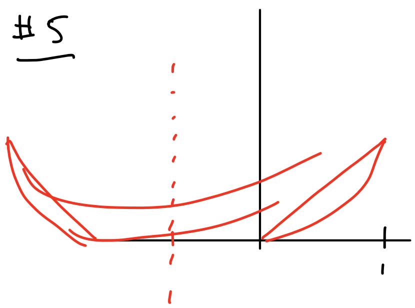
$= \frac{x}{2}+3$

$$V=\pi\int_2^4 \left(\frac{x}{2}+3\right)^2-2^2\,dx = \pi\int_2^4 \frac{x^2}{4}+3x+9-4\,dx$$

$$= \pi\int_2^4 \frac{x^2}{4}+3x+5\,dx = \pi\left(\frac{1}{12}x^3+\frac{3}{2}x^2+5x\right)\Big|_2^4$$

$$= \pi\left(\frac{1}{12}\cdot 4^3+\frac{3}{2}\cdot 4^2+5\cdot 4\right) - \pi\left(\frac{1}{12}\cdot 2^3+\frac{3}{2}\cdot 2^2+5\cdot 2\right)$$

#5



$$2\pi\int_0^1 (x+1)(x-x^2)\,dx$$

$$= 2\pi\int_0^1 \cancel{x^2}+x-x^3-\cancel{x^2}\,dx$$

$$= 2\pi\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)\Big|_0^1 = 2\pi\left(\frac{1}{2}-\frac{1}{4}\right) = \pi$$

#6 a

$$f(x) = \ln\left((2x-3)^2\right)$$

$$f'(x) = \frac{1}{(2x-3)^2}\cdot 2(2x-3)\cdot 2$$

$$f'(2) = \frac{1}{(4-3)^2}\cdot 2(4-3)\cdot 2 = 4$$

b

$$3+e^{7x+2} = 4 \qquad x = -2/7$$

$$e^{7x+2} = 1$$

$$\ln e^{7x+2} = \ln 1$$

$$7x+2 = 0$$

#7

a $\quad -\sin(e^{3x^2+x}) \cdot e^{3x^2+x} \cdot (6x+1)$

b $\quad \int_0^{\pi} \sin x \; e^{5+\cos x} \, dx$

$u = 5+\cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= -\int e^u \, du = -e^u = -e^{5+\cos x} \Big|_0^{\pi}$$

$$= -e^{5+\cos\pi} - -e^{5+\cos 0}$$

$$= -e^4 + e^6$$

#8

a $\quad \frac{d}{dx} \ln(3+4^{x^2}) = \frac{1}{3+4^{x^2}} \cdot 4^{x^2} \ln 4 \cdot 2x$

b $\quad \log_2 16 = 4$

$\log 100 = 2$

$\log_4 40$ is impossible by hand

$\log_9 \frac{1}{3} = -\frac{1}{2}$

9 $\quad \int_1^2 \frac{1}{\sqrt{4-x^2}} \, dx = \int_1^2 \frac{1}{\sqrt{4}\sqrt{1-\frac{x^2}{4}}} \, dx$

$$= \frac{1}{2} \int_1^2 \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \, dx$$

$u = x/2$

$du = \frac{1}{2} dx$

$2du = dx$

$$= \cancel{\frac{1}{2}} \int \frac{1}{\sqrt{1-u^2}} \cdot \cancel{2} du = \arcsin u$$

$$= \arcsin \frac{x}{2} \Big|_1^2 = \arcsin \frac{2}{2} - \arcsin \frac{1}{2}$$

$$= \arcsin 1 - \arcsin \frac{1}{2} = \pi/2 - \pi/6$$

10 $\lim_{x\to\infty} \frac{\ln x}{x^{1/2}}$ $\frac{\infty}{\infty}$

$$\overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{1}{x} \cdot 2x^{1/2} = \lim_{x\to\infty} 2\frac{1}{x^{1/2}} \to \frac{1}{\infty}$$

$$= 0$$

11 $\int (2-x^2)e^{4x}\,dx$ $u = 2-x^2$ $du = -2x\,dx$

$dv = e^{4x}dx$ $v = \frac{1}{4}e^{4x}$

$$= (2-x^2)\cdot\frac{1}{4}e^{4x} - \int \frac{1}{4}e^{4x}\cdot -2x\,dx$$

$u = x$ $du = dx$

$$= (2-x^2)\cdot\frac{1}{4}e^{4x} + \frac{1}{2}\int xe^{4x}\,dx$$

$dv = e^{4x}dx$ $v = \frac{1}{4}e^{4x}$

$$= (2-x^2)\cdot\frac{1}{4}e^{4x} + \frac{1}{2}\left(x\cdot\frac{1}{4}e^{4x} - \int\frac{1}{4}e^{4x}\,dx\right)$$

$$= (2-x^2)\cdot\frac{1}{4}e^{4x} + \frac{1}{2}\left(x\cdot\frac{1}{4}e^{4x} - \frac{1}{4}\cdot\frac{1}{4}e^{4x}\right) + C$$