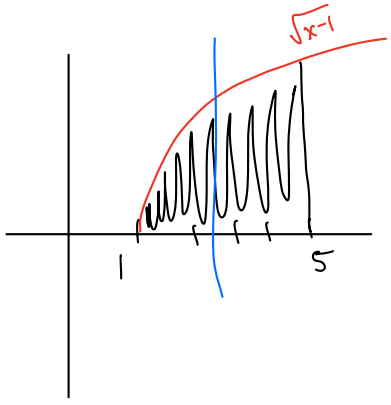
Math 1172 Homework #2

Section 5.2 # 13, 21
5.3 # 2, 25

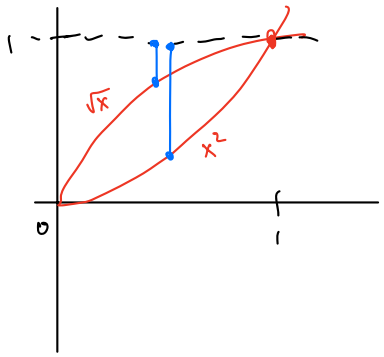
5.2 # 13 Region bounded by $y = \sqrt{x-1}$, $y = 0$, $x = 5$, revolved around x-axis.



$$\pi \int_1^5 (\sqrt{x-1})^2 \, dx = \pi \int_1^5 x-1 \, dx = \pi \left(\frac{x^2}{2} - x \right) \Big|_1^5$$

$$= \pi \left(\frac{5^2}{2} - 5 \right) - \pi \left(\frac{1^2}{2} - 1 \right)$$

5.2 # 21 Region bounded by $y = x^2$ & $x = y^2$ ($y = x^{1/2}$), revolved around $y = 1$.



$$V = \pi \int_0^1 (1-x^2)^2 - (1-x^{1/2})^2 \, dx$$

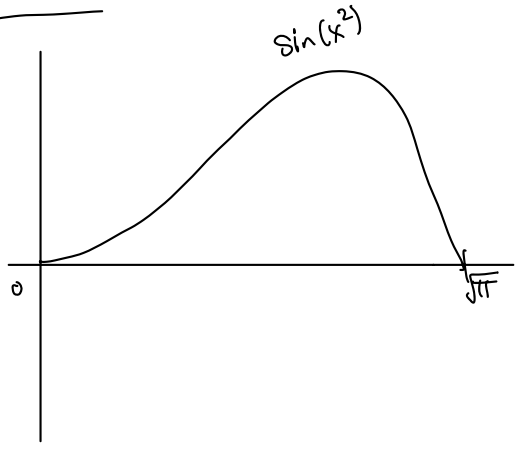
$$= \pi \int_0^1 (1 - 2x^2 + x^4) - (1 - 2x^{1/2} + x) \, dx$$

$$= \pi \int_0^1 -2x^2 + x^4 + 2x^{1/2} - x \, dx$$

$$= \pi \left(-\frac{2}{3}x^2 + \frac{1}{5}x^5 + 2 \cdot \frac{2}{3} x^{3/2} - \frac{1}{2}x^2 \right) \Big|_0^1$$

$$= \pi \left(-\frac{2}{3} \cdot 1 + \frac{1}{5} \cdot 1 + \frac{4}{3} \cdot 1 - \frac{1}{2} \cdot 1 \right) - \pi (0)$$

5.3 #2



$$V = 2\pi \int_0^{\sqrt{\pi}} x \sin(x^2)\, dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

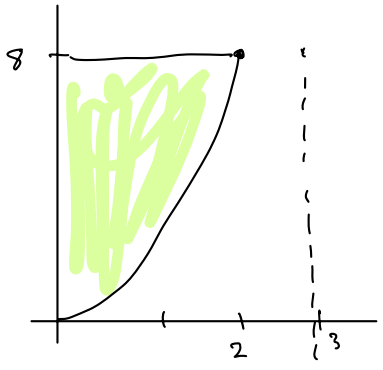
$$= 2\pi \int \sin u \cdot \frac{1}{2} du$$

$$= \pi \int \sin u \, du = \pi \cdot (-\cos u)$$

$$= -\pi \cos(x^2) \Big|_0^{\sqrt{\pi}} = -\pi \cos(\sqrt{\pi}^2) - (-\pi \cos 0)$$

$$= -\pi \cos \pi + \pi = -\pi(-1) + \pi = 2\pi$$

5.3 #25 Region bounded by $y = x^3$, $y = 8$, $x = 0$, revolved about $x = 3$.



$$V = 2\pi \int_2^0 (x-3)(8 - x^3)\, dx$$

$$= 2\pi \int_2^0 8x - 24 - x^4 + 3x^3\, dx$$

$$= 2\pi \left(4x^2 - 24x - \frac{1}{5}x^5 + \frac{3}{4}x^4 \right) \Big|_2^0$$

$$= 2\pi \left(4 \cdot 2^2 - 24 \cdot 2 - \frac{1}{5} \cdot 2^5 + \frac{3}{4} \cdot 2^4 \right) - 0$$