

Math 1172 HW #6

Section 7.2 # 11, 18

7.3 # 10

7.4 # 12

7.2 # 11

$$\int_0^{\pi/2} \sin^2 x \cos^2 x dx$$

$$= \int_0^{\pi/2} \frac{1}{2} (1 - \cos 2x) \cdot \frac{1}{2} (1 + \cos 2x) dx$$

$$= \frac{1}{4} \int_0^{\pi/2} 1 - \cos^2 2x dx = \frac{1}{4} \int_0^{\pi/2} 1 - \frac{1}{2} (1 + \cos 4x) dx$$

$$= \frac{1}{4} \int_0^{\pi/2} \frac{1}{2} - \frac{1}{2} \cos 4x dx = \frac{1}{4} \left(\frac{1}{2} x - \frac{1}{2} \cdot \frac{1}{4} \sin 4x \right) \Big|_0^{\pi/2}$$

$$= \frac{1}{4} \left(\frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{8} \sin(4 \cdot \frac{\pi}{2}) \right) - \frac{1}{4} \left(\frac{1}{2} \cdot 0 - \frac{1}{8} \cdot 0 \right)$$

$$= \frac{1}{4} \left(\frac{\pi}{4} - \frac{1}{8} \cdot 0 \right) - 0 = \frac{\pi}{16}$$

7.2 # 18

$$\int \tan^2 x \cos^3 x dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \cos^3 x dx = \int \sin^2 x \cos x dx$$

$$u = \sin x$$

$$du = \cos x dx$$

$$= \int u^2 du = \frac{1}{3} u^3 + C = \frac{1}{3} \sin^3 x + C$$

7.3 #10

$$\int \frac{x^2}{\sqrt{9-x^2}} dx$$

$$x = 3 \sin \theta$$

$$dx = 3 \cos \theta d\theta$$

$$\theta = \sin^{-1} \frac{x}{3}$$

$$= \int \frac{9 \sin^2 \theta}{\sqrt{9-9\sin^2 \theta}} 3 \cos \theta d\theta = \int \frac{9 \sin^2 \theta}{\sqrt{9\cos^2 \theta}} 3 \cos \theta d\theta$$

$$= \int \frac{9 \sin^2 \theta}{\cancel{3 \cos \theta}} \cancel{3 \cos \theta} d\theta = 9 \int \sin^2 \theta d\theta$$

$$= 9 \int \frac{1}{2} (1 - \cos 2\theta) d\theta = \frac{9}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) + C$$

$$= \frac{9}{2} \left(\sin^{-1} \frac{x}{3} - \frac{1}{2} \sin \left(2 \cdot \sin^{-1} \frac{x}{3} \right) \right) + C$$

7.4 #12

$$\int_0^1 \frac{x-4}{x^2-5x+6} dx$$

$$\frac{x-4}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$= \frac{A(x-3) + B(x-2)}{\dots}$$

$$x-4 = Ax-3A+Bx-2B$$

$$A+B=1 \quad B=1-A$$

$$-3A-2B=-4 \rightarrow -3A-2(1-A)=-4$$

$$-3A-2+2A=-4$$

$$-A=-2 \quad A=2 \quad B=1-2=-1$$

$$\int_0^1 \frac{x-4}{x^2-5x+6} dx = \int_0^1 \frac{2}{x-2} - \frac{1}{x-3} dx = 2 \ln|x-2| - \ln|x-3| \Big|_0^1$$

$$= 2 \ln|1-2| - \ln|1-3| - (2 \ln|-2| - \ln|-3|)$$

