Math 1172 Homework # 11

Section 11.8 # 15, 26

Section 11.9 # 10, 17

11.8 # 15

$$\sum \frac{x^n}{n^4 \cdot 4^n}$$

Ratio: $$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^4 \cdot 4^{n+1}} \cdot \frac{n^4 \cdot 4^n}{x^n} \right| = \lim_{n\to\infty} |x| \cdot \left(\frac{n}{n+1}\right)^4 \cdot \frac{1}{4}$$

$$= \frac{|x|}{4}$$

ROC: $\frac{|x|}{4} < 1$ $\quad |x| < 4$ $\quad$ ROC $= 4$

$-4 < x < 4$

endpts:

$x = -4$: $\sum \frac{(-4)^n}{n^4 \cdot 4^n} = \sum \frac{(-1)^n}{n^4}$ converges by alt. series test.

$x = 4$: $\sum \frac{\cancel{4^n}}{n^4 \cdot \cancel{4^n}} = \sum \frac{1}{n^4}$ converges (p-series, $p > 1$)

So IOC $= [-4, 4]$

11.8 #26 $\sum \frac{(2x-1)^n}{5^n \sqrt{n}}$

ratio:

$$L = \lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}} \cdot \frac{5^n \sqrt{n}}{(2x-1)^n} \right| = \lim_{n\to\infty} |2x-1| \cdot \frac{1}{5} \cdot \left(\frac{n}{n+1}\right)^{1/2}$$

$$= \frac{|2x-1|}{5}$$

ROC: $\frac{|2x-1|}{5} < 1$

$|2x-1| < 5$

$|x-1/2| < 5/2$ ROC $= 5/2$

$-5/2 < x - 1/2 < 5/2$

$-2 < x < 3$

endpts

$x = -2$

$$\sum \frac{(2\cdot(-2)-1)^n}{5^n \sqrt{n}} = \sum \frac{(-5)^n}{5^n \sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}}$$ conv. by alt. series test.

$x = 3$

$$\sum \frac{(2\cdot 3-1)^n}{5^n \sqrt{n}} = \sum \frac{5^n}{5^n \sqrt{n}} = \sum \frac{1}{n^{1/2}}$$ div. (p-series, $p \leq 1$)

So IOC $= (-2, 3]$

11.9 #10

$$\frac{x}{2x^2+1} = x \cdot \frac{1}{1-(-2x^2)}$$

$$= x \sum_{n=0}^{\infty} (-2x^2)^n = \sum (-2)^n x^{2n+1} = \sum (-1)^n 2^n x^{2n+1}$$

conv when $|-2x^2| < 1$

i.e. $2|x^2| < 1$

$|x^2| < 1/2$

$|x| < 1/\sqrt{2}$

$IOC = (-1/\sqrt{2}, 1/\sqrt{2})$

11.9 #17

$$\frac{x}{(1+4x)^2} = x \cdot \sum \frac{1}{(1+4x)^2}$$

use $\sum \frac{1}{(1+x)^2} = \sum (-1)^{n+1} n x^{n-1}$

conv. when $|x| < 1$.

$$= x \sum n \cdot (-1)^{n+1} (4x)^{n-1}$$

$$= x \sum n \cdot (-1)^{n+1} \cdot 4^{n-1} x^{n-1}$$

$$= \sum n \cdot (-1)^{n+1} 4^{n-1} x^n$$

Converges when $|4x| < 1$, i.e. $|x| < 1/4$.

$IOC = (-1/4, 1/4)$ $ROC = 1/4$