Math 1172 Midterm

#1 a

$$\int_0^{\pi/3} 6x + \cos x \, dx = 6 \cdot \tfrac{1}{2}x^2 + \sin x \Big|_0^{\pi/3}$$

$$= 3(\pi/3)^2 + \sin \pi/3 - (3 \cdot 0^2 + \sin 0)$$

$$= \pi^2/3 + \frac{\sqrt{3}}{2}$$

#1b

$$\int x^2(x+1)^2 dx = \int x^2(x^2+2x+1)\, dx$$

$$= \int x^4 + 2x^3 + x^2 = \frac{1}{5}x^5 + \frac{2}{4}x^4 + \frac{1}{3}x^3 + C$$

 # 2

$$\int 7x^3(3x^4-2)^5 dx$$

$u = 3x^4 - 2$

$du = 12x^3\, dx$

$\frac{1}{12} du = x^3 dx$

$$= 7\int u^5 \cdot \frac{1}{12} du = \frac{7}{12}\int u^5\, du$$

$$= \frac{7}{12} \cdot \frac{1}{6}u^6 + C = \frac{7}{12 \cdot 6}(3x^4-2)^6 + C$$

 # 3

Find intersections:

$x = y^2 - 2y$

$x = y$

$y^2 - 2y = y$

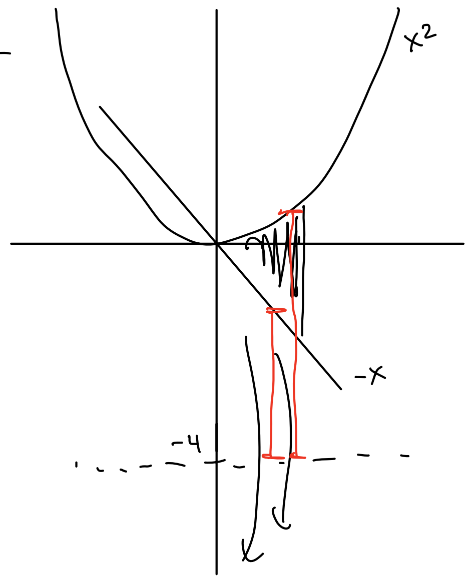
$y^2 - 3y = 0$

$y(y-3) = 0$ $\qquad \boxed{y=0, \quad y=3}$

$$A = \int_0^3 y - (y^2 - 2y)\, dy = \int_0^3 3y - y^2\, dy$$

$$= \frac{3}{2}y^2 - \frac{1}{3}y^3 \Big|_0^3 = \frac{3}{2}\cdot 3^2 - \frac{1}{3}\cdot 3^3 - \left(\frac{3}{2}\cdot 0^2 - \frac{1}{3}\cdot 0^3\right)$$

#4



inner $= -x - (-4) = 4 - x$

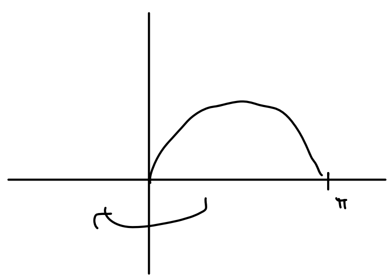
outer $= x^2 - (-4) = x^2 + 4$

$$V = \pi \int_0^2 (x^2+4)^2 - (4-x)^2\, dx = \pi \int_0^2 x^4 + 8x^2 + 16 - (16 - 8x + x^2)\, dx$$

$$= \pi \int_0^2 x^4 + 7x^2 + 8x\, dx = \pi\left(\frac{1}{5}x^5 + \frac{7}{3}x^3 + 4x^2\right)\Big|_0^2$$

$$= \pi\left(\frac{1}{5}\cdot 2^5 + \frac{7}{3}\cdot 2^3 + 4\cdot 2^2\right) - \pi(0)$$

#5



$$2\pi \int_0^\pi x \sin x\, dx \qquad u = x \quad du = dx \qquad dv = \sin x\, dx \quad v = -\cos x$$

$$= 2\pi\left(x(-\cos x) - \int_0^\pi -\cos x\, dx \right)$$

$$= 2\pi(-x\cos x + \sin x)\Big|_0^\pi$$

$$= 2\pi(-\pi \overset{-1}{\cos\pi} + \overset{0}{\sin\pi}) - 2\pi(0\cos 0 + \overset{0}{\sin 0})$$

$$= 2\pi \cdot \pi = 2\pi^2$$

#6

a

$$e^{x^2-7x} \cdot \cos x + \sin x \; e^{x^2-7x} (2x-7)$$

b

$$\int_0^4 3x\, e^{3-x^2}\, dx$$

$u = 3 - x^2$

$du = -2x\, dx$

$-\frac{1}{2} du = x\, dx$

$$3 \int e^u \cdot \frac{-1}{2} du = -\frac{3}{2} \int e^u = \frac{-3}{2} e^u$$

$$= \frac{-3}{2} e^{3-x^2} \Big|_0^4 = \frac{-3}{2} e^{3-16} - \frac{-3}{2} e^3$$

#7

a

$$x^2 \cdot \frac{1}{x^3+5} \cdot 3x^2 + \ln(x^3+5) \cdot 2x$$

b

$$\int \frac{e^x}{2e^x - 3}\, dx$$

$u = 2e^x - 3$

$du = 2e^x\, dx$

$\frac{1}{2} du = e^x\, dx$

$$= \int \frac{1}{u} \cdot e^x dx = \int \frac{1}{u} \cdot \frac{1}{2} du = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|2e^x - 3| + C$$

#8 a

$$\frac{\cos x \cdot 2^x \ln 2 - 2^x \cdot -\sin x}{\cos^2 x}$$

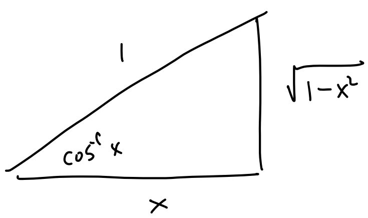
b i $\log_4 64 = 3$ since $4^3 = 64$

ii $\log 1000 = 3$ since $10^3 = 1000$

iii $\log_4 2 = 1/2$ since $4^{1/2} = 2$

iv $\log_8 16$ impossible since 16 isn't a nice power of 8.

#9



$\sin(\cos^{-1} x) = \sqrt{1-x^2}$

$\cos(\cos^{-1} x) = x$

$\tan(\cos^{-1} x) = \dfrac{\sqrt{1-x^2}}{x}$

#10

$$\lim_{x\to\infty} x^2 e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x} \to \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2x}{e^x} \to \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2}{e^x} = 0$$

#11

$$\int_2^5 x^4 \ln x \, dx$$

$u = \ln x$ $\quad du = 1/x \, dx$

$dv = x^4 dx$ $\quad v = \frac{1}{5}x^5$

$$= \ln x \cdot \frac{1}{5}x^5 - \int \frac{1}{5}x^5 \cdot \frac{1}{x}\, dx$$

$$= \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4 \, dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\cdot\frac{1}{5}x^5 \Big|_2^5$$

$$= \frac{1}{5}\cdot 5^5 \ln 5 - \frac{1}{25}\cdot 5^5 - \left(\frac{1}{5}\cdot 2^5 \ln 2 - \frac{1}{25}\cdot 2^5\right)$$

#12

$$\int \sin^5 x \cos^3 dx = \int \sin^5 x \cos^2 x \cdot \cos x \, dx$$

$$= \int \sin^5 x \left(1 - \sin^2 x\right) \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int u^5 (1 - u^2) \, du$$

$$= \int u^5 - u^7 \, du = \frac{1}{6}u^6 - \frac{1}{8}u^8 + C$$

$$= \frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C$$

$\underline{\#13}$

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx \qquad x = 3\sin\theta \qquad dx = 3\cos\theta\,d\theta \qquad \theta = \sin^{-1}\frac{x}{3}$$

$$= \int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}}\; 3\cos\theta\,d\theta$$

$$= \int \frac{9\sin^2\theta}{\cancel{3}\,\cancel{\sqrt{\cos^2\theta}}}\; \cancel{3\cos\theta}\,d\theta = 9\int \sin^2\theta\,d\theta$$

$$= 9\int \frac{1}{2}(1-\cos 2\theta)\,d\theta = \frac{9}{2}\int 1-\cos 2\theta \;\; d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{9}{2}\left(\sin^{-1}\frac{x}{3} - \frac{1}{2}\sin\left(2\sin^{-1}\frac{x}{3}\right)\right) + C$$

$\underline{\#14}$

$$\int \frac{x^3-2x^2-10x-19}{x^2-4x-5}$$

$$\begin{array}{r|l} & x+2 \\ \hline x^2-4x-5 & x^3-2x^2-10x-19 \\ & \underline{x^2-4x^2-5x} \\ & 2x^2-5x-19 \\ & \underline{2x^2-8x-10} \\ & 3x-9 \end{array}$$

$$= \int x+2+\frac{3x-9}{x^2-4x-5}\;dx$$

partial frax: $\dfrac{3x-9}{(x-5)(x+1)} = \dfrac{A}{x-5} + \dfrac{B}{x+1}$

$$3x-9 = A(x+1) + B(x-5)$$

$$3x-9 = (A+B)x + A - 5B$$

$A+B=3$ $\quad$ $B=3-A$

$A-5B=-9$ $\quad$ $A-5(3-A)=-9$

$A-15+5A=-9$

$6A=6$

$A=1$ $\quad$ $B=2$

$$\int x+2+\frac{1}{x-5}+\frac{2}{x+1}\,dx$$

$$=\frac{1}{2}x^2+2x+\ln|x-5|+2\ln|x+1|+C$$

15

$$\int_0^\infty x\,3^{-x^2}dx=\lim_{t\to\infty}\int_0^t x\,3^{-x^2}dx$$

$u=-x^2$

$du=-2x\,dx$

$-\frac{1}{2}du=x\,dx$

$$=\lim_{t\to\infty}\int_0^t 3^u\cdot-\frac{1}{2}du$$

$$=-\frac{1}{2}\lim_{t\to\infty}\int 3^u du=\frac{-1}{2}\lim_{t\to\infty}\frac{1}{\ln 3}3^u$$

$$=\frac{-1}{2\ln 3}\lim_{t\to\infty}3^{-x^2}\Big|_0^t$$

$$=\frac{-1}{2\ln 3}\cdot\lim_{t\to\infty}\left(3^{-t^2}-3^0\right)$$

$$=\frac{-1}{2\ln 3}\lim_{t\to\infty}\left(\underbrace{\frac{1}{3^{t^2}}}_{0}-1\right)$$

$$=\frac{-1}{2\ln 3}\cdot-1=\frac{1}{2\ln 3}$$

16

$$f(x) = 1 + \frac{x^{3/2}}{3}$$

$$f'(x) = \frac{1}{2} x^{1/2}$$

$$\int_1^4 \sqrt{1 + (f'(x))^2}\, dx = \int_1^4 \sqrt{1 + \tfrac{1}{4}x}\; dx$$

$u = 1 + \frac{1}{4}x$

$du = \frac{1}{4}\, dx$

$$= \int \sqrt{u} \cdot 4\, du = 4 \int u^{1/2}\, du$$

$$= 4 \cdot \frac{2}{3} u^{3/2} = \frac{8}{3} \left(1 + \tfrac{1}{4}x\right)^{3/2} \Big|_1^4$$

$$= \frac{8}{3}(1+1)^{3/2} - \frac{8}{3}\left(1 + \tfrac{1}{4}\right)^{3/2}$$