Math 1172 Homework #6

Section 7.2 #4, 11

7.3 #10, 12

Section 7.2 #4

$$\int_0^{\pi/4} \sin^5 x\,dx = \int_0^{\pi/4} \sin^4 x \quad \sin x\,dx$$

$$= \int_0^{\pi/4} (1-\cos^2 x)^2 \sin x\,dx$$

$u = \cos x$
$du = -\sin x\,dx$

$$= \int (1-u^2)^2 \cdot -du = -\int 1 - 2u^2 + u^4\,du$$

$$= -\left(u - \tfrac{2}{3}u^3 + \tfrac{1}{5}u^5 \right)$$

$$= -\left(\cos x - \tfrac{2}{3}\cos^3 x + \tfrac{1}{5}\cos^5 x \right)\Big|_0^{\pi/4}$$

$$= -\left(\cos \pi/4 - \tfrac{2}{3}\cos^3 \pi/4 + \tfrac{1}{5}\cos^5 \pi/4 \right) - -\left(\cos 0 - \tfrac{2}{3}\cos^3 0 + \tfrac{1}{5}\cos^5 0 \right)$$

$$= -\left(\frac{1}{\sqrt{2}} - \frac{2}{3}\cdot\left(\frac{1}{\sqrt{2}}\right)^3 + \frac{1}{5}\left(\frac{1}{\sqrt{2}}\right)^5 \right) - -\left(1 - \frac{2}{3} + \frac{1}{5} \right)$$

7.2 # 11

$$\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$$

$$= \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x) \cdot \frac{1}{2}(1 + \cos 2x) \, dx$$

$$= \frac{1}{4} \int_0^{\pi/2} 1 - \cos^2 2x \, dx = \frac{1}{4} \int_0^{\pi/2} 1 - \frac{1}{2}(1 + \cos 4x) \, dx$$

$$= \frac{1}{4} \int_0^{\pi/2} 1 - \frac{1}{2} - \frac{1}{2}\cos 4x \, dx = \frac{1}{4} \int_0^{\pi/2} \frac{1}{2} - \frac{1}{2}\cos 4x \, dx$$

$$= \frac{1}{4}\left(\frac{1}{2}x - \frac{1}{2} \cdot \frac{1}{4}\sin 4x \right)\Bigg|_0^{\pi/2}$$

$$= \frac{1}{4}\left(\frac{1}{2} \cdot \pi/2 - \frac{1}{8}\sin(4 \cdot \pi/2) \right) - \cancel{\frac{1}{4}\left(\frac{1}{2} \cdot 0 - \frac{1}{8}\sin 0 \right)}^{0}$$

$$= \frac{1}{16}\pi - \frac{1}{32}\cancel{\sin 2\pi}^{0} = \pi/16$$

7.3 #10

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx$$

$x = 3\sin\theta$

$dx = 3\cos\theta$

$$= \int \frac{(3\sin\theta)^2}{\sqrt{9-(3\sin\theta)^2}} \cdot 3\cos\theta\,d\theta$$

$$= \int \frac{9\sin^2\theta}{3\sqrt{1-\sin^2\theta}} \cdot 3\cos\theta\,d\theta = 9\int \frac{\sin^2\theta}{\sqrt{\cos^2\theta}}\cos\theta\,d\theta$$

$$= 9\int \sin^2\theta\,d\theta = 9\int \tfrac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$x = 3\sin\theta$

$\theta = \sin^{-1}\frac{x}{3}$

$$= \frac{9}{2}\left(\sin^{-1}\frac{x}{3} - \frac{1}{2}\sin 2\left(\sin^{-1}\frac{x}{3}\right)\right) + C$$

7.3 #12

$$\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$$

$x = 6\sin\theta$

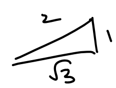
$dx = 6\cos\theta\,d\theta$

$x=0 \rightarrow \theta=0$

$x=3$

$\rightarrow 3 = 6\sin\theta$

$\sin\theta = 1/2$



$\theta = \pi/6$

$$= \int_0^{\pi/6} \frac{6\sin\theta}{\sqrt{36-(6\sin\theta)^2}} \cdot 6\cos\theta\,d\theta$$

$$= \int_0^{\pi/6} \frac{\cancel{6}\sin\theta}{\cancel{6}\sqrt{\cancel{1-\sin^2\theta}}} \cdot 6\cancel{\cos\theta}\,d\theta = 6\int_0^{\pi/6}\sin\theta\,d\theta$$

$$= 6(-\cos\theta)\Big|_0^{\pi/6} = 6\left(-\cos\tfrac{\pi}{6}\right) - 6\cdot(-\cos 0)$$

$$= 6\cdot -\frac{\sqrt{3}}{2} + 6\cdot 1$$