Math 1172 Homework #8

Section 7.8 #42

8.1 #13, 16, 51

7.8 #42

$$\int_0^5 \frac{w}{w-2}\,dw \qquad \text{discont. at } 2$$

$$\lim_{t\to 2} \int_0^t \frac{w}{w-2}\,dw + \lim_{t\to 2} \int_2^5 \frac{w}{w-2}\,dw$$

First one: $\displaystyle \lim_{t\to 2} \int_0^2 \frac{w}{w-2}\,dw$ $\qquad u = w-2 \qquad w = u+2$

$du = dw$

$$= \lim_{t\to 2} \int \frac{u+2}{u}\,du = \lim_{t\to 2} \int 1 + \frac{2}{u}\,du = \lim_{t\to 2} u + 2\ln|u|$$

$$= \lim_{t\to 2} w-2 + 2\ln|w-2| \Big|_0^t$$

$$= \lim_{t\to 2} t-2 + 2\ln|t-2| - \left(0-2+2\ln|0-2|\right)$$

$\downarrow$

$\ln 0$

diverges!

8.1 # 13

$$y = \frac{1}{3}x^3 + \frac{1}{4} \cdot x^{-1}$$

$$f'(x) = x^2 - \frac{1}{4}x^{-2}$$

$$\int_1^2 \sqrt{1 + \left(x^2 - \frac{1}{4}x^{-2}\right)^2}\, dx = \int_1^2 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16}x^{-4}}\, dx$$

$$= \int_1^2 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16}x^{-4}}\, dx = \int_1^2 \sqrt{\left(x^2 + \frac{1}{4}x^{-2}\right)^2}\, dx$$

$$= \int_1^2 x^2 + \frac{1}{4}x^{-2}\, dx = \frac{1}{3}x^3 - \frac{1}{4}x^{-1} \Big|_1^2$$

$$= \frac{1}{3} \cdot 2^3 - \frac{1}{4} \cdot 2^{-1} - \left(\frac{1}{3} \cdot 1^3 - \frac{1}{4} \cdot 1^{-1}\right)$$

8.1 # 16

$$y = \ln(\cos x)$$

$$f'(x) = \frac{1}{\cos x} \cdot -\sin x = -\tan x$$

$$\int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx = \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} \sec x\, dx = \ln\left|\sec x + \tan x\right| \Big|_0^{\pi/3}$$

$$= \ln\left|\sec \pi/3 + \tan \pi/3\right| - \ln\left|\sec 0 + \tan 0\right|$$

8.1 #51

$$f(x) = \frac{1}{4}e^{x} + e^{-x}$$

$$f'(x) = \frac{1}{4}e^{x} - e^{-x}$$

Area under curve $= \displaystyle\int_a^b \frac{1}{4}e^{x} + e^{-x}\,dx$

Arclength $= \displaystyle\int_a^b \sqrt{1 + \left(\frac{1}{4}e^{x} - e^{-x}\right)^2}\,dx = \int_a^b \sqrt{1 + \frac{1}{16}e^{2x} - \frac{1}{2} + e^{-2x}}\,dx$

$$= \int_a^b \sqrt{\frac{1}{16}e^{2x} + \frac{1}{2} + e^{-2x}}\,dx = \int_a^b \sqrt{\left(\frac{1}{4}e^{x} + e^{-x}\right)^2}\,dx$$

$$= \int_a^b \frac{1}{4}e^{x} + e^{-x}\,dx$$

Same!