Math 1172 HW #11

Section 11.3 #17, 19

Section 11.4 #17, 19

11.3 #17

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2} \to \int_1^{\infty} \frac{\sqrt{x}+4}{x^2}\,dx = \int_1^{\infty} (x^{1/2}+4)x^{-2}\,dx$$

$$= \int_1^{\infty} x^{-3/2} + 4x^{-2} = \lim_{t\to\infty} -2x^{-1/2} - 4x^{-1} \Big|_1^t$$

$$= \lim_{t\to\infty} -2t^{-1/2} - 4t^{-1} - \left(-2\cdot 1^{-1/2} - 4\cdot 1^{-1}\right)$$

$$= 0 - 0 - (\cdots) \quad \text{converges}$$

$$\text{so} \quad \sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2} \quad \text{converges.}$$

11.3 #19

$$\sum_{n=1}^{\infty} \frac{1}{n^2+4} \to \int_1^{\infty} \frac{1}{x^2+4}\,dx = \frac{1}{4}\int_1^{\infty} \frac{1}{\frac{x^2}{4}+1}$$

$$= \frac{1}{4}\int_1^{\infty} \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx \qquad \begin{aligned} u &= x/2 \\ du &= \tfrac{1}{2}\,dx \\ 2du &= dx \end{aligned}$$

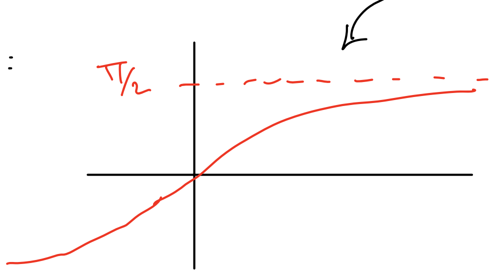
$$= \frac{1}{4} \lim_{t\to\infty} \int \frac{1}{u^2+1}\cdot 2du = \lim_{t\to\infty} \frac{1}{2} \arctan u$$

$$= \lim_{t\to\infty} \frac{1}{2}\arctan\frac{x}{2}\Big|_1^t = \lim_{t\to\infty} \frac{1}{2}\arctan\frac{t}{2} - \frac{1}{2}\arctan\frac{1}{2}$$

$$= \frac{1}{2}\cdot \pi/2 - \frac{1}{2}\arctan 1/2$$

converges

arctan looks like:



So $\sum \frac{1}{n^2+4}$ converges

11.4 #17

$$\sum_{n=1}^{\infty} \frac{1+\cos n}{e^n} \leq \sum \frac{1+1}{e^n} = \sum 2 \cdot \left(\frac{1}{e}\right)^n$$

geometric with $r = 1/e < 1$
converges

So the bigger one converges,

so $\sum \frac{1+\cos n}{e^n}$ converges.

11.4 #19

$\sum \frac{4^{n+1}}{3^n - 2}$ limit comp. with $\frac{4^{n+1}}{3^n}$

$$\lim_{n\to\infty} \frac{\cancel{4^{n+1}}}{3^n-2} \cdot \frac{3^n}{\cancel{4^{n+1}}} = \lim_{n\to\infty} \frac{3^n}{3^n-2}$$

$$= \lim_{n\to\infty} \frac{3^n/3^n}{3^n/3^n - 2/3^n} = \lim_{n\to\infty} \frac{1}{1-2/3^n} = \frac{1}{1-0} = 1$$

So $\sum \frac{4^{n+1}}{3^n-2}$ & $\sum \frac{4^{n+1}}{3^n}$ have same convergence.

$$\sum \frac{4^{n+1}}{3^n} = \sum 4 \cdot \frac{4^n}{3^n} = \sum 4 \cdot \left(\frac{4}{3}\right)^n$$

geometric with $r = 4/3$, so it <u>diverges</u>

So $\displaystyle\sum \frac{4^{n+1}}{3^n - 2}$ <u>diverges</u>