Math 1172 HW # 13

11.6 # 22, 30
11.8 # 6, 15

11.6 #22

$$\sum \frac{(-2)^n}{n^n}$$ root test!

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(-2)^n}{n^n}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^n}} = \lim_{n\to\infty} \frac{2}{n} = 0$$

$L < 1$, so it converges

11.6 #30

$$\sum_{n=1}^{\infty} \frac{n\,5^{2n}}{10^{n+1}}$$ ratio test!

$$L = \lim_{n\to\infty} \left| \frac{(n+1)\,5^{2(n+1)}}{10^{n+1+1}} \cdot \frac{10^{n+1}}{n\,5^{2n}} \right|$$

$$= \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{10^{n+1}}{10^{n+2}} \cdot \frac{5^{2n+2}}{5^{2n}}$$

$$= \lim_{n\to\infty} \frac{1+1/n}{1} \cdot \frac{1}{10} \cdot 5^2 = \frac{25}{10}$$

$L > 1$, so it diverges.

11.8 #6 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

Ratio test:

$$L = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{(-1)^n x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^{n+1}}{x^n} \cdot \frac{n^{1/3}}{(n+1)^{1/3}} \right| = \lim_{n\to\infty} |x| \cdot \left(\frac{n}{n+1}\right)^{1/3}$$

$$= |x|$$

Converges when $|x| < 1$, ie. $-1 < x < 1$.

Check endpts:

$x = 1$ $\displaystyle\sum \frac{(-1)^n}{\sqrt[3]{n}}$ converges (alt. series test)

$x = -1$ $\displaystyle\sum \frac{(-1)^n (-1)^n}{\sqrt[3]{n}} = \sum \frac{1}{\sqrt[3]{n}} = \sum \frac{1}{n^{1/3}}$

diverges (p-series, $p \le 1$)

So IOC $= (-1, 1]$

ROC $= 1$

11.8 #15 $\quad \sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

$$L = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^4 \cdot 4^{n+1}} \cdot \frac{n^4 \cdot 4^n}{x^n} \right|$$

$$= \lim_{n\to\infty} \frac{n^4}{(n+1)^4} \cdot \frac{4^n}{4^{n+1}} \cdot \left| \frac{x^{n+1}}{x} \right|$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^4 \cdot \frac{1}{4} \cdot |x| = \frac{1}{4}|x|$$

converges when $\frac{1}{4}|x| < 1$, i.e. $-1 < \frac{1}{4}x < 1$

$$-4 < x < 4$$

check endpts:

$x=4$ $\quad \sum \frac{4^n}{n^4 4^n} = \sum \frac{1}{n^4}$ converges. (p-series)

$x=-4$ $\quad \sum \frac{(-4)^n}{n^4 4^n} = \sum \frac{(-1)^n}{n^4}$ converges (alt. series test)

So IOC is $[-4, 4]$

ROC is 4.