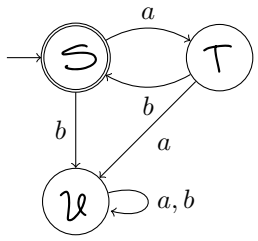
Math 3342 Exam #1

Question 1. (15 points) For the following DFA, please find a formula for all strings which are accepted, and then prove that they are accepted.



$L(M) = \{(ab)^n\}$

Thm $\delta^*(S, (ab)^n) = S$

Pf Induction on n

Base case $n=0$ WTS $\delta^*(S, (ab)^0) = S$

$\delta^*(S, (ab)^0) = \delta^*(S, \varepsilon) = S$ ✓

inductive case Assume $\delta^*(S, (ab)^k) = S$

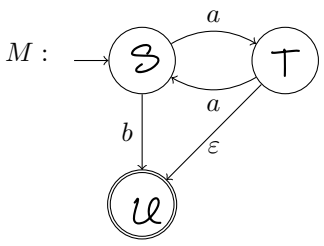
WTS $\delta^*(S, (ab)^{k+1}) = S$

We have $\delta^*(S, (ab)^{k+1}) = \delta^*(S, (ab)^k ab)$

$= \delta^*(\delta^*(S, (ab)^k), ab)$

$= \delta^*(S, ab) = S$ Thm.

This whole page is about this NFA which I'll call M:



Question 2. (8 points) Please find $L(M)$. You can describe it in words, or write a set-theory formula for it.

$L(M) = \{a^{2n}b\} \cup \{a^{2n+1}\}$

Question 3. (5 points) Please give the formal description of M.

$M = (\{S,T,U\}, \{a,b\}, \delta, S, \{U\})$

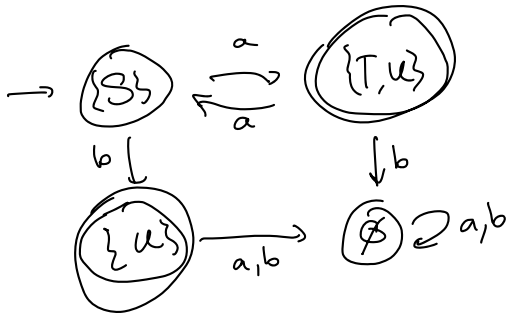
where:

$\delta(S,a) = \{T\}$ $\quad \delta(T,a) = \{S\}$ $\quad \delta(U,a) = \emptyset$

$\delta(S,b) = \{U\}$ $\quad \delta(T,b) = \emptyset$ $\quad \delta(U,b) = \emptyset$

$\delta(S,\varepsilon) = \emptyset$ $\quad \delta(T,\varepsilon) = \{U\}$ $\quad \delta(U,\varepsilon) = \emptyset$

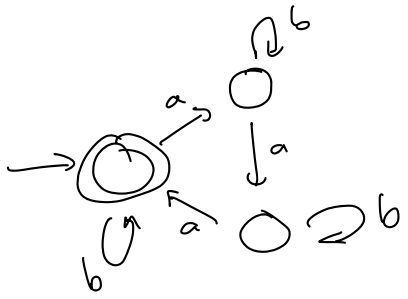
Question 4. (12 points) Please use the subset construction to create a DFA equivalent to M.



Question 5. (12 points each) In each part, draw a DFA or NFA (your choice) for the given language. Use a DFA for at least one of your answers, and use an NFA for at least one of your answers. Each time, the alphabet should be $\{a, b\}$.

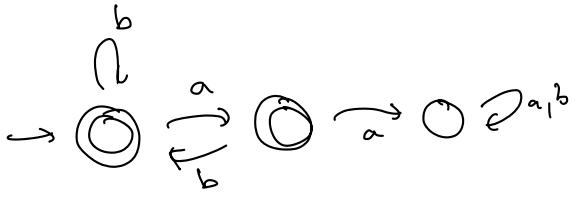
a) $\{x \in \{a, b\}^* \mid$ the number of a's in x is divisible by 3$\}$

DFA

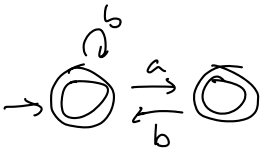


b) $\{x \in \{a, b\}^* \mid x$ never uses 2 consecutive as$\}$

DFA

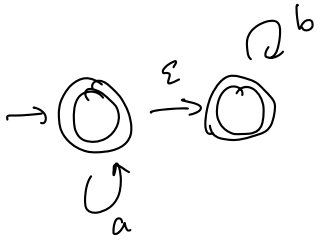


NFA



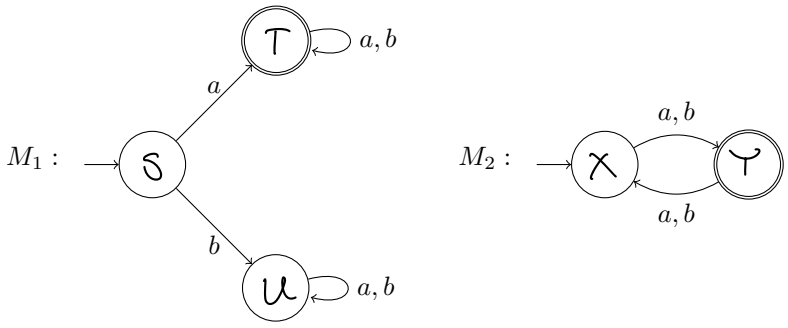
c) $\{x \in \{a, b\}^* \mid x$ has no a after any $b$$\}$

NFA

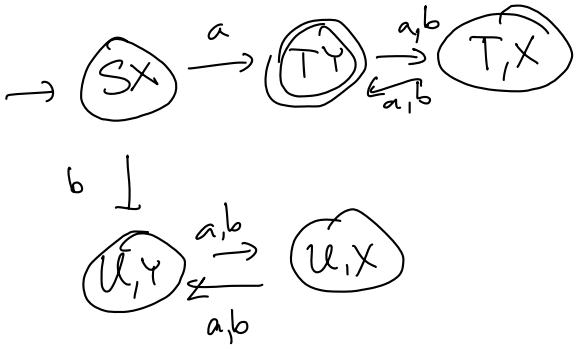


DFA

This whole page is about these two DFAs:



Question 6. (12 points) Please give a DFA for $L(M_1) \cap L(M_2)$ using one of the constructions that we discussed in class.



Question 7. (12 points) Please give a NFA for $L(M_1) \cap L(M_2)$ by first describing $L(M_1) \cap L(M_2)$ as a set (in words, or with formulas), and then making an NFA for that set.

$L(M_1) \cap L(M_2)$ is all strings starting with a and odd length.

NFA: