

Math 3342 Exam #2

Question 1. (6 points each) In each part, please give a regular expression for the given language:

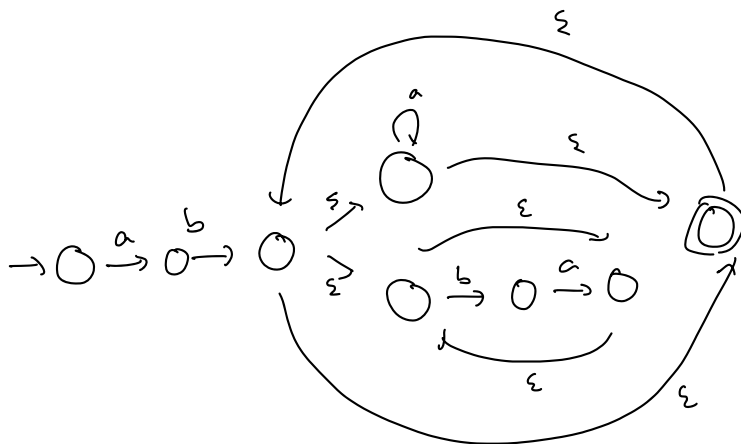
- a) $\{xbaby \in \{a,b\}^* \mid x \text{ has even length and } y \text{ is any string}\}$

$$\left((a+b)(a+b) \right)^* b a b (a+b)^*$$

- b) $\{a^n b^k a b^m \in \{a,b\}^* \mid k < 3 \text{ and } m > 3\}$

$$a^* (\varepsilon + b + b^2) a b^4 b^*$$

Question 2. (10 points) Please give an NFA for this regular expression: $ab(a^* + (ba)^*)^*$



Question 3. (7 points each) In each part, give a context free grammar for the given language. One of them is not possible, so for that one, just write "impossible." (You don't need to prove that it's impossible.)

a) $\{a^n b^k (ab)^k b^n \mid n, k \in \mathbb{N}\}$

$$\begin{aligned} S &\rightarrow a S b \mid T \\ T &\rightarrow b T a b \mid \epsilon \end{aligned}$$

b) $\{x a x^R \mid x \in \{a, b\}^*\}$

$$S \rightarrow a S a \mid b S b \mid a$$

c) $\{a^n b^n a^n b^n \mid n \in \mathbb{N}\}$

impossible

d) The set of all strings consisting of one or more blocks of either all *as* or all *bs*, separated by hyphens. (The hyphens will be produced as part of your grammar.) So a typical string in this language looks like:

$$aaa - bbb - a - bb - aaaaaa - b$$

(Each block should be non-empty. Make sure there is no hyphen on the end.)

$$\begin{aligned} S &\rightarrow B \mid B - S \\ B &\rightarrow X \mid Y \\ X &\rightarrow a X \mid a \\ Y &\rightarrow b Y \mid b \end{aligned}$$

Question 4. (7 points) Please give a regular expression for the language in part d) above.

$$((a^+ \mid b^+) -)^* (a^+ \mid b^+)$$

Question 5. (7 points each) In each part, say if the language is regular or not. If it's regular, give a regular expression for the language. If not, prove that it is not regular.

a) $\{aba^n b^n \mid n \in \mathbb{N}\}$

Not regular:

$$\text{Let } D_i = \frac{d}{d a b a^i} L = \{a^{n-i} b^n\}$$

these are all different, so

b) $\{a^n b^m c^k \mid n, m, k \in \mathbb{N}\}$

regular: $a^* b^* c^*$

c) $\{xax \mid x \in \{a, b\}^*\}$

not regular:

$$\text{if } x = a^i y, \\ \text{then } xax = a^i y a a^i y$$

$$\text{let } D_i = \frac{d}{d a^i} L = \{y a^{i+1} y\}$$

these are all different.

Question 6. This whole page is about this stack machine:

read	pop	push
a	S	TX
b	T	S
ε	S	ε
a	X	ε

a) (5 points) Please give a derivation showing that $ababaa$ is accepted on the stack machine.

$$\begin{aligned}
 (ababaa, S) &\mapsto (babaa, TX) \mapsto (abaa, SX) \mapsto (baa, TXX) \\
 &\mapsto (aa, SXX) \mapsto (aa, XX) \mapsto (a, X) \mapsto (\varepsilon, \varepsilon) \quad \checkmark
 \end{aligned}$$

b) (5 points) Please give a CFG equivalent to this stack machine, and show a grammar derivation for $ababaa$.

$$S \rightarrow aTX \mid \varepsilon$$

$$T \rightarrow bS$$

$$X \rightarrow a$$

$$S \Rightarrow aTX \Rightarrow abSX \Rightarrow$$

$$\Rightarrow abaTXX \Rightarrow ababSXX$$

$$\Rightarrow ababXX \Rightarrow ababaa$$

c) (6 points) Please find the language of the stack machine. Describe it either as a set or in words.

→ simplifies to:

$$S \rightarrow abSa \mid \varepsilon$$

$$\{ (ab)^n a^n \}$$

d) (6 points) What if we keep everything else the same, but replace the first rule with: read a , pop S , push SX ? How does that change the language of the machine?

In this case we never get any T on the stack,
so no string containing b can ever be accepted.

It's as above, but with all b 's omitted.

$$\{ a^n a^n \} = \{ a^{2n} \}$$