

Math 3342

Homework #2

Chapter 2 #3, #6c

Extra probs #1, #3

Chapter 2 #3

a) $\{x \in \{a,b\}^+ \mid |x| \text{ is odd}\}$

b) $\{x \in \{a,b\}^+ \mid |x| \text{ is even}\}$

c) \emptyset

d) $\{x \in \{a,b\}^+ \mid |x| > 0\}$

e) $\{x \in \{a,b\}^+ \mid x \text{ has an odd \# of } b\text{'s}\}$

Chapter 2 #6c

$(\Sigma \{A,B,C\}, \{a,1\}, \delta, A, \{A,C\})$

where:

$\delta(A,0) = B$

$\delta(C,0) = C$

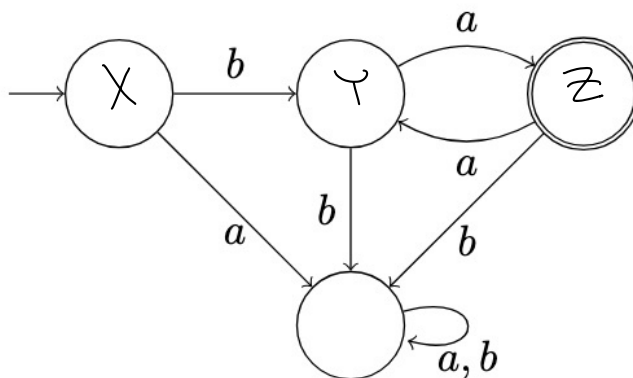
$\delta(A,1) = C$

$\delta(C,1) = C$

$\delta(B,0) = B$

$\delta(B,1) = B$

Extra problems #1



$L(M) = \{ba^{2n+1}\}$ formally:

Then $\delta^*(X, ba^{2n+1}) = Z$

PF by induction on n

base case $n=0$: WTS $\delta^*(X, ba^0) = Z$, and this is true.

induction case Assume $\delta^*(X, ba^{2k+1}) = Z$
WTS $\delta^*(X, ba^{2(k+1)+1}) = Z$.

We have:
$$\begin{aligned} \delta^*(X, ba^{2(k+1)+1}) &= \delta^*(X, ba^{2k+1}aa) \\ &= \delta^*(\delta^*(X, ba^{2k+1}), aa) \\ &\stackrel{IH}{=} \delta^*(Z, aa) = Z \end{aligned}$$
 Shown!

#3 $L(M) = \{(ab)^n\}$

Thm $\delta^*(X, (ab)^n) = X$

PF induction on n

base case $n=0$ WTS $\delta^*(X, (ab)^0) = X$.

We have $\delta^*(X, (ab)^0) = \delta^*(X, \epsilon) = X$ ✓

induction Assume $\delta^*(X, (ab)^k) = X$
WTS $\delta^*(X, (ab)^{k+1}) = X$.

We have $\delta^*(X, (ab)^{k+1}) = \delta^*(X, (ab)^k ab)$

$= \delta^*(\delta^*(X, (ab)^k), ab)$

$= \delta^*(X, ab) = X$

Shown!

