Homework #11

Question 1. In each part, give a Turing machine for the given language.

a) $\{x\#\bar{x} \mid x \in \{0,1\}^*\}$ where # is a special separator symbol written on the tape (treat it like any other letter), and the bar means "not", so $\bar{0} = 1$ and $\bar{1} = 0$.

b) $\{xcycx \mid x, y \in \{a,b\}^*\}$.

Question 2. In each part, give a Turing machine which computes the given function.

a) The "head" function, which takes a nonempty string on the alphabet $\{a,b\}^*$ and returns just its first letter.

b) The "tail" function, which takes a nonempty string on the alphabet $\{a,b\}^*$ and returns the whole thing except the first letter.

c) For $x \in \{a,b\}^*$, the function $f(x) = a^{|x|}$.

d) $f(a^n) = a^{2n}$. (One possible strategy you could use: first turn the string a^n into something like $\bar{a}^n b^n$ by marking each a and writing b at the end to match it. Then run across the whole string and turn everything into an a, resulting in a^{2n}.)

e) $f(a^{2n}) = a^n$. (You can use a similar strategy: mark an a at the beginning, and then blank one out at the end. Then when you're done, change every $\bar{a}$ back to an a.)

f) For binary numbers, $f(x) = x + 4$. (You can assume that x is large enough so that it already uses at least 3 digits.)

g) For binary numbers, $f(x) = x \div 4$. (Give only the quotient as the answer, ignoring any remainders.)

h) For binary numbers, $f(x) = x \mod 4$.